Physics-based Deformable Modeling of Volumes and Surfaces for Medical Image Registration, Segmentation and Visualization

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Foreword

This book summarizes the main results of the research work I carried out during my Ph.D. thesis in the Communications and Remote Sensing Laboratory (TELE lab) at the Université catholique de Louvain. I started my thesis in October 1997, and spent the academic year 1998-1999 in Boston, where I worked both at the Massachusetts Institute of Technology in Professor Dewey’s laboratory and at Harvard Medical School in Professor Kikinis’s laboratory. Financial support for this work was initially granted by the Belgian “Fonds pour la Formation à la Recherche dans l’Industrie et l’Agriculture” (FRIA) and over the last six months by the Région Wallonne government in Belgium through the EMIM-2 project. Besides this necessary financial support, there are a number of people without whose contribution and support this work would never have been as it is or even existed.

Firstly I would like to express my sincere gratitude to my principal supervisor, Benoît Macq, without whose continued support the work presented here would not have been possible. He “bribed” me into joining the lab to work on medical image processing, which was not such a bad deal. He helped me out whenever he could, gave me the freedom to direct my research the way I wanted, and allowed me to peak outside the lab. This clearly expanded my horizon and led to very fruitful collaborations with other researchers from all over the world. Financial support for traveling to meetings, conferences and workshops has never been a problem, which I believe is a luxury for a researcher, and I therefore must acknowledge him for his great management.

The environment in the TELE lab was also greatly enhanced by the means he put at our disposal: I am thinking of the great graphics workstation I worked on, but also the systems administrators Pascal Maes and Frederic Oger who were always available to fix our most horrible mistakes, and the
secretary Patricia Focant who managed to satisfy all my requests when arranging my travels. Beyond these practical aspects, I will always recall the great atmosphere in room A-157, first with Olivier Cuisenaire and Vincent Darmstaedter, and after I returned from Boston with DJ “Huguolin” Dorban and “Sylbanito” Jaume. The time we shared working in the same room has been very enjoyable both personally and scientifically.

Secondly, I am also extremely grateful to Professor Dewey for having hosted me during a year in his lab at MIT (Massachusetts Institute of Technology, Cambridge, USA). It was a wonderful experience both from a personal and professional point of view. At MIT, he allowed me to meet with some of the world’s greatest researchers in finite element analysis, mesh generation and computer vision. The environment in his lab was also very stimulating and lead to very interesting discussions and exchanges mainly with Patrick McCormick and Yuan Cheng.

Simon Warfield started following and supervising my work quite closely when I arrived in Boston. He very quickly accepted to follow up with me after I first met him at the Surgical Planning Laboratory (Brigham and Women’s Hospital, Harvard Medical School, Boston, USA). It has been a fantastic scientific stimulation to the advance of my research. The numerous discussions we had not only about my research but also about related problems gave me a great insight on what medical image processing and analysis actually are about. He spent hours with me trying to debug my programs, and much of my coding style derives from his expertise. Even after I returned from Boston, Simon has been a continuing and stimulating support in my research. I don’t think I ever had to wait longer than a day before Simon would get back to my e-mails.

The thesis jury provided me with very valuable comments, and I have to thank them all for their feedback on the manuscript as well as for the very fruitful discussion we had during the private defence.

The necessary “real life” and personal support of the last few years was amply supported by my wife Katia. She endured lonely evenings and weekends while I was working, trying to meet some impossible dead-lines or attending meetings abroad. I should also mention sleepless nights, endless complaints that I was never going to make it, etc. Her support really boosted my confidence whenever work pressure was exceeding what I could
personnaly take up.

Finally, I also need to thank my parents who have steadily supported my choices throughout my studies.

Matthieu Ferrant
April 2001.
Contents

1 Introduction 1
  1.1 Context of the Work .......................... 1
  1.2 Targetted Applications .......................... 3
  1.3 Proposed Method and Main Contributions of this Thesis 4
  1.4 Organization of the Text ......................... 7

I Models and Algorithms 9

2 Finite Element Modeling of Elastic Membranes and Volumes 11
  2.1 Mathematical Formulation ......................... 11
  2.2 Implementation Considerations ....................... 17
  2.3 Illustration of Volume Deformation .................. 19

3 Finite Element Mesh Generation 21
  3.1 Motivation ........................................ 21
  3.2 Surface Mesh Generation ............................ 22
    3.2.1 Review of Existing Methods ....................... 22
  3.3 Volumetric Mesh Generation ......................... 24
    3.3.1 Review of Existing Methods ....................... 24
    3.3.2 Our Approach .................................. 27
  3.4 Description of our Algorithm ....................... 28
    3.4.1 Consistent Tetrahedralization of Hexahedra ......... 28
    3.4.2 Construction of a Multi-Resolution Tetrahedralization 30
    3.4.3 Tetrahedralization of Prisms and Pyramids .......... 30
    3.4.4 Accurate Representation Boundary Surfaces .......... 31
  3.5 Results on Synthetic Images ....................... 31
  3.6 Mesh Quality Assessment ........................... 34
3.7 Discussion .................................................. 35

4 Deformable Surface Modeling ....................... 37
  4.1 Description of our Algorithm ....................... 37
  4.2 Initialization of the Deformable Surface .......... 39
  4.3 Results on Synthetic Images ....................... 41
    4.3.1 Sphere to Cube Experiment ..................... 41
    4.3.2 Sphere to Paraboloid Matching ................. 42

5 Volumetric Elastic Image Matching ............... 43
  5.1 Motivation ............................................. 43
  5.2 Mathematical Formulation .......................... 44
  5.3 Synthetic Experiments .............................. 46
    5.3.1 Growing Sphere .................................. 47
    5.3.2 Matching a Sphere onto a Cube ................. 47
  5.4 Discussion ............................................ 48

6 Using Deformable Surface Models with Volumetric Models 51
  6.1 Motivation ............................................. 51
  6.2 Description of our Method .......................... 52
  6.3 Results on Synthetic Image Sequences .......... 53
    6.3.1 Embedded Translated Cube Experiment .......... 53
    6.3.2 Embedded Sphere to Ellipsoid Experiment .... 54
  6.4 Discussion ............................................ 57

II Applications ........................................... 59

7 Volumetric Elastic Image Matching : Results on Medical Data 61
  7.1 Volumetric Elastic Matching Results .............. 61
    7.1.1 Arm Data ....................................... 61
    7.1.2 Ventricular Matching ............................ 62
    7.1.3 Lesion Growth .................................... 62
    7.1.4 Brain Shift Analysis ............................. 63
  7.2 Discussion ............................................ 64

8 Deformable Atlas-Based Segmentation of MR Images of the Brain 67
  8.1 Introduction ......................................... 67
    8.1.1 Volumetric Methods .............................. 69


<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1.2</td>
<td>Surface-based Methods</td>
<td>69</td>
</tr>
<tr>
<td>8.1.3</td>
<td>Our Approach</td>
<td>70</td>
</tr>
<tr>
<td>8.2</td>
<td>Atlas Matching Algorithm</td>
<td>71</td>
</tr>
<tr>
<td>8.2.1</td>
<td>Initialization of the Deformable Surfaces - Global Atlas Registration</td>
<td>71</td>
</tr>
<tr>
<td>8.2.2</td>
<td>Deformable Surface Matching</td>
<td>71</td>
</tr>
<tr>
<td>8.2.3</td>
<td>Volumetric Atlas Matching</td>
<td>71</td>
</tr>
<tr>
<td>8.3</td>
<td>Results</td>
<td>72</td>
</tr>
<tr>
<td>8.4</td>
<td>Conclusion</td>
<td>74</td>
</tr>
<tr>
<td>9</td>
<td>Deformable Registration of Intraoperative MR Images of the Brain</td>
<td>77</td>
</tr>
<tr>
<td>9.1</td>
<td>Introduction</td>
<td>77</td>
</tr>
<tr>
<td>9.1.1</td>
<td>Context and Motivation</td>
<td>77</td>
</tr>
<tr>
<td>9.1.2</td>
<td>Review of Related Work</td>
<td>79</td>
</tr>
<tr>
<td>9.1.3</td>
<td>Proposed Method</td>
<td>82</td>
</tr>
<tr>
<td>9.2</td>
<td>Algorithms</td>
<td>82</td>
</tr>
<tr>
<td>9.2.1</td>
<td>Preoperative and Intraoperative Segmentation</td>
<td>87</td>
</tr>
<tr>
<td>9.2.2</td>
<td>Modeling Tissue Resection</td>
<td>88</td>
</tr>
<tr>
<td>9.3</td>
<td>Results</td>
<td>89</td>
</tr>
<tr>
<td>9.3.1</td>
<td>Material Properties</td>
<td>89</td>
</tr>
<tr>
<td>9.3.2</td>
<td>Registration of Two Intraoperative MR Images of the Brain</td>
<td>89</td>
</tr>
<tr>
<td>9.3.3</td>
<td>Serial Registration of Intraoperative MR Images of the Brain</td>
<td>94</td>
</tr>
<tr>
<td>9.3.4</td>
<td>Timeline Analysis of Deformable Registration</td>
<td>107</td>
</tr>
<tr>
<td>9.3.5</td>
<td>Discussion</td>
<td>108</td>
</tr>
<tr>
<td>9.4</td>
<td>Conclusions and Perspectives</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>Conclusions and Perspectives</td>
<td>113</td>
</tr>
<tr>
<td>10.1</td>
<td>Major Achievements and Contributions</td>
<td>113</td>
</tr>
<tr>
<td>10.2</td>
<td>Perspectives and Future Work</td>
<td>114</td>
</tr>
<tr>
<td>10.3</td>
<td>Related Publications</td>
<td>116</td>
</tr>
<tr>
<td>10.4</td>
<td>Software Distribution and Collaborations</td>
<td>117</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

This chapter outlines the context of the work, and presents the applications it is targeting. The proposed methods are briefly presented, along with the major contributions they have lead to both from an algorithmic and applicative point of view. Finally, an overview of all the chapters of the thesis is given.

1.1 Context of the Work

During the past decades, medical image processing and analysis have evolved very quickly, and are raising new challenges every day for scientists and engineers. It has become a discipline of its own, that still retains common aspects with classical image processing and analysis, but which has developed its own specificities.

The special nature of medical images derives from their method of acquisition as well as from the subjects from which images are being acquired. Medical imaging has been distinguished primarily by its ability to provide information about the volume beneath the surface of objects. Indeed, images are obtained for medical purposes almost exclusively to probe the otherwise invisible anatomy below the skin. This information may be in the form of the two-dimensional projections acquired by traditional radiography, the two-dimensional slices of ultrasound, or full three-dimensional (3D) mappings, such as those provided by computed tomography (CT), magnetic resonance imaging (MRI), single photon emission computed tomography (SPECT), positron emission tomography (PET), functional MRI (fMRI), 3D ultrasound (US) or digital subtraction angiography (DSA) (Sonka and
FitzPatrick, 2000).

The fact that medical image processing and analysis deal mostly with living bodies brings a number of major differences in comparison to computer or robot vision. The objects of interest are soft and deformable with three-dimensional shapes whose surfaces are rarely rectangular, cylindrical or spherical, and whose features rarely include the planes or straight lines that are so frequent in common engineering applications. These shapes can vary considerably from one individual to another, sometimes as much as normal and pathological shape differ. The available knowledge about human anatomy also represents a major difference with most classical computer vision problems, where no prior assumption about the images exists.

Medical image processing distinguishes itself by the development of problem-specific approaches to the enhancement of raw medical data for the purposes of selective visualization, modeling, characterization, and further analysis. Medical image analysis then concentrates on the development of techniques to supplement the mostly qualitative and frequently subjective assessment of medical images by human experts with a variety of new information that is quantitative, objective, and reproducible.

The development and use of physics-based modeling (PBM) methods and techniques makes it possible to address successfully difficult problems that are not possible with purely geometric and phenomenological techniques. PBM methods associate geometry, dynamics and material properties in order to model physical objects and their interactions with the physical world. Therefore, as opposed to purely geometric models, physics-based models incorporate additional constraints (e.g. material properties) that are very useful in modeling, simulation and estimation applications (Metaxas, 1997). Medical image processing and analysis can benefit from physics-based approaches, as the objects depicted in the images always have a physics-based behavior.

In medical imaging applications, it is often desirable to estimate, track, or characterize complex shapes and motions of internal organs such as the brain or the heart. In such applications, deformable PBM provide the appropriate mechanism for modeling tissue properties, estimating, characterizing and visualizing the motion of these organs, and for extracting model parameters which can be useful for diagnosis, surgical planning, and
1.2 Targetted Applications

Recent years have witnessed a tremendous growth in the use of image-guided surgery (IGS) systems. After the acquisition of pre-operative imaging, such systems allow surgeons to visualize and navigate into the anatomy of the patient in the operation room (OR) during surgery. One key element of such systems is the registration of the pre-operative imaging. The registration module provides a mechanism for transforming the pre-operative imaging into the intra-operative coordinate frame of the patient in the OR. Usually, only rigid body transformations (rotation and translation) are considered in these systems, but in many cases, other dynamic and non-rigid deformations occur during surgery, therefore causing inaccuracies of the surgical planning with respect to the pre-operative imaging it is based on in the IGS system.

In the case of image-guided neurosurgery, problems have arisen because of brain deformation during surgery. During neurosurgery, the brain has been observed to dynamically shift and deform significantly both when opening the skull and when resecting tumor. It is therefore of crucial importance for
neurosurgeons to be able to track and characterize this deformation during surgery. The use of intraoperative MR imaging is a key technology in this task that allows us to track the shape changes of the brain during surgery. This will help us in designing better models and to propose a PBM with minimal external input. A typical setting of image-guided neurosurgery with an intraoperative MR scanner is shown in Figure 1.1.

1.3 Proposed Method and Main Contributions of this Thesis

The main focus in this thesis has been on the development of a PBM capable of tracking and characterizing the phenomenon of brain shift during neurosurgery.

Figure 1.2: Coronal view of successive intraoperative MR scans illustrating brain deformation during neurosurgery. a) Before start of surgery, b) after opening of the dura, c) after start of tumor resection.

Figure 1.2 illustrates the physics-based deformation the brain undergoes during surgery using intraoperative MR imaging. The first view shows the brain before surgery starts, the second one shows the brain sinking in the direction of gravity after opening of the dura, and the third one shows the brain after tumor resection has started. The resected areas show up as black holes within the brain tissue. The issue here is to track the shape changes of the brain given the information intraoperative imaging provides.

1 Courtesy of Ron Kikinis, Surgical Planning Laboratory, Brigham and Woman’s Hospital and Harvard Medical School, Boston, MA02115 USA.
The discretization of PBM problems using the Finite Element (FE) Method and elasticity is becoming more and more popular for various applications such as surgical simulation and surgical planning (Delingette, 1998; Skrinar et al., 1998; Paulsen et al., 1999), because soft tissue deformation can often be approximated using linear elasticity. The FE method, in conjunction with an elastic deformation model, is often chosen for its reliable behavior and accuracy as compared to simpler analogies such as mass-spring models (e.g. Gibson (1997)) and models only computing the deformation at the surface (e.g. Bro-Nielsen (1997)).

There are basically two kinds of approaches for tracking and characterizing soft tissue deformation for medical applications: those using a model and trying to simulate its behavior with estimated forces such as gravity, and those using external input, i.e. imaging to drive the deformation of the model (e.g. Metaxas (1997)). All models proposed so far for modeling brain shift, both the simulation ones and the image-based ones have only been used to the deformation between two time points, e.g. pre- and post surgery, or pre- and during surgery.

Therefore, for our purpose—tracking and characterizing brain shift—, FE modeling of linear elasticity is preferred over simplified models, and because we would like to be able to dynamically track and characterize the deformation of the brain, we use image sequences so as to be able to provide the FE model with sufficient input so that it can reflect and characterize the deformations it is undergoing.

Our idea has been to model the objects that are to be tracked in the image sequence and to deform them using boundary deformation information we extract from the images. We track the boundary deformation of key objects in the image sequence using an active surface algorithm, and we use this deformation field as an input to deform the volumetric FE biomechanical model representing the objects in the initial image.

Because the algorithm can be seen as a generic tool for warping an image onto another one, this technique can be used for the application for which we developed it (deformable brain registration) but also for other applications such as deformable brain atlas matching. The algorithm then serves as a tool for the automatic localization and segmentation of anatomical structures.
Within the elastic FE modeling framework, we also exploit volumetric elastic image matching, for which elasticity serves as a regularization of a purely image-based similarity criterion between images. The equations of the problem are discretized using the FE method, which has advantages over the common finite differences approach.

The use of discrete images, such as those provided by scanners, imposes a discrete approximation on any model used to represent the objects in the image. Since the direct use of the image samples within the finite element discretization of the model would lead to prohibitive memory storage and computational load, a model with elements covering several image samples needs to be designed. We have therefore developed a new tetrahedral mesh generation algorithm for labeled 3D image data.

The major outcome of this thesis is an integrated PBM approach for the dynamic tracking and characterization of soft tissue deformation from image sequences. Our approach combines ideas and methods from continuum mechanics, computer graphics and image processing, but it still is closer to the purely image-based approaches than to those doing simulations without external input.

The main contributions of this work are

- a volumetric biomechanical simulation and deformation model that uses surface displacements extracted from image sequences,
- a tetrahedral mesh generation algorithm for image volumes with adaptable size control which is fast and accurate,
- an optimized FE deformation model that makes the execution of the algorithm fast enough to be used within the time constraints of image-guided surgery,
- the demonstration that such an approach in conjunction with intra-operative imaging is suitable to track and characterize brain shift during neurosurgery, as well as to update preoperative imaging so that it reflects shape changes due to neurosurgery.
From an applicative point of view:

- significant contributions to the serial analysis of brain deformation during neurosurgery using the previously mentioned algorithm. More specifically, to our knowledge, this is the first time tumor resection and brain swelling afterwards are analysed using a biomechanical model in conjunction with intraoperative imaging.

1.4 Organization of the Text

In this work, we present a generic physics-based framework and algorithms for modeling the behavior and the interaction of medical structures from image sequences depicting motion of internal organs. The ultimate aim is the tracking and the characterization of brain shift during neurosurgery, but other applications and related algorithms are also presented. The first part of the thesis (Chapters 2 to 6) concentrates on the models and algorithms, while the second part (Chapters 7 to 9) elaborates on medical applications of the previously described models and algorithms.

Chapter 2 presents the mathematical equations needed to model elastic bodies, and how we solve the model equations within a Finite Element (FE) discretization framework. Some implementation considerations are also discussed in this chapter. An example of the behavior of our model is described on a synthetic example with different model parameters.

Chapter 3 describes how we generate FE discretization models from 3D image volumes, both for surfaces and volumetric bodies. Examples of tetrahedral mesh generation from synthetic image data are shown.

Chapter 4 describes how we use our generic FE implementation of elastic membranes for solving active surface problems. We propose a solution to effectively initialize active surfaces for improved convergence and accuracy. We describe our algorithm, and give some examples illustrating the behavior of our algorithm on synthetic images.

Chapter 5 presents an approach to 3D elastic image matching within a FE discretization framework. In this chapter, we propose to adapt the constitutive equations modeling elastic volumes presented in Chapter 2 to incorporate an image similarity constraint. The behavior of the algorithm...
on synthetic image data is illustrated by a few experiments.

**Chapter 6** describes an approach for modeling biomechanical deformations in image sequences from boundary deformations. We explain how a deformable surface algorithm such as presented in Chapter 4 can be used in conjunction with a volumetric FE model such as presented in Chapter 2 to produce a volumetric deformation field for matching an image onto another one. Results on synthetic images are presented to illustrate the algorithm.

**Chapter 7** presents medical applications for which we use the algorithm presented in Chapter 5. Images showing arm exercise, ventricular enlargement and lesion growing in Multiple Sclerosis (MS), and brain shift are used to illustrate the behavior of this algorithm on medical data.

**Chapter 8** illustrates how we use the algorithm presented in Chapter 6 to solve atlas matching problems. Some preliminary results are presented on MR images of the brain.

**Chapter 9** describes how we use the concepts presented in Chapter 6 to register intraoperative MR images of the brain with preoperative image data for improved image-guided neurosurgery. We also explain how the algorithm can be used for the registration of serial intraoperative MR images of the brain. The issue of tumor removal and topology change in the model is also addressed in this chapter. Results of registration and characterization of the deformation obtained on actual sequences depicting brain shift are presented.

**Chapter 10** draws the conclusions of this work and reviews the contributions of the thesis and its applications. We also point out perspectives for further research. Finally, we summarize the related publications and collaborations this thesis has lead to.
Part I

Models and Algorithms
Chapter 2

Finite Element Modeling of Elastic Membranes and Volumes

In this chapter, we introduce the theory and the way we set up equation systems for solving Finite Element modeling of elastic volumes and surfaces. Implementation considerations are discussed, and an example of the behavior of our implementation with different parameters is shown.

2.1 Mathematical Formulation

Assuming a linear elastic continuum with no initial stresses or strains, the deformation energy of an elastic body submitted to externally applied forces can be expressed as (Zienkewicz and Taylor, 1987)\(^1\)

\[
E = \frac{1}{2} \int_{\Omega} \sigma^T \epsilon \, d\Omega + \int_{\Omega} F \cdot u \, d\Omega \tag{2.1}
\]

where \(u = u(x)\) is the displacement vector, \(F = F(x)\) the vector representing the forces applied to the elastic body (forces per unit volume, surface forces or forces concentrated at the nodes), and \(\Omega\) the body on which one is working (where \(u(x) = (u_x(x), u_y(x), u_z(x))\) and \(x = (x, y, z)\) in 3D space). \(\epsilon\) is the strain tensor, which can be written as a vector for notation

\(^1\)Superscript T designs the transpose of a vector or a matrix
simplicity:

\[ \epsilon = \left( \epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \right)^T \]

\[ = \left( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}, \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}, \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \right)^T \]

\[ = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \end{pmatrix} u = Lu \] (2.2)

\[ \sigma = \left( \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx} \right)^T = D\epsilon \] (2.3)

and \( \sigma \) the stress tensor, linked to the strain tensor by the constitutive equations of the material. In the case of linear elasticity, with no initial stresses or strains, using a vector notation for \( \sigma \) and \( \epsilon \) this relation is described as

In the case of an orthotropic material, the material has three mutually perpendicular planes of elastic symmetry (Ting, 1996). Hence there are three kinds of material parameters:

- the Young moduli \( E_i \) relate tension and the stretch in the main orthogonal directions,
- the shear moduli \( G_{ij} \) relate tension and stretch in other directions than those of the planes of elastic symmetry,
- the Poisson ratios \( \nu_{ij} \) represent the ratio of the lateral contraction due to longitudinal stress in a given plane.
2.1 Mathematical Formulation

If the material’s main orthogonal directions coincide with the coordinate axes, one has:

\[
\begin{align*}
E_x &= \frac{\partial \sigma_x}{\partial \epsilon_x} \quad E_y &= \frac{\partial \sigma_y}{\partial \epsilon_y} \quad E_z &= \frac{\partial \sigma_z}{\partial \epsilon_z} \\
G_{xy} &= \frac{\partial \tau_{xy}}{\partial \gamma_{xy}} \quad G_{yz} &= \frac{\partial \tau_{yz}}{\partial \gamma_{yz}} \quad G_{zx} &= \frac{\partial \tau_{zx}}{\partial \gamma_{zx}} \\
\nu_{xy} &= -\frac{\partial \epsilon_y}{\partial \epsilon_x} \quad \nu_{yz} &= -\frac{\partial \epsilon_z}{\partial \epsilon_y} \quad \nu_{zx} &= -\frac{\partial \epsilon_z}{\partial \epsilon_x}
\end{align*}
\]

Thus for an orthotropic material there are nine unknown parameters. The elasticity matrix then becomes:

\[
D = \frac{1}{\Delta}
\begin{pmatrix}
\frac{1-v_{xy}v_{yz}}{E_y E_z} & \frac{\nu_{yz}+v_{xy}v_{yz}}{E_y E_z} & \nu_{xz}+v_{xy}v_{yz} & 0 & 0 & 0 \\
\cdot & \frac{1-v_{xy}v_{yz}}{E_x E_z} & \frac{\nu_{yz}+v_{xy}v_{yz}}{E_x E_z} & 0 & 0 & 0 \\
\cdot & \cdot & \frac{1-v_{xy}v_{yz}}{E_y E_z} & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & G_{xy} \Delta & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & G_{yz} \Delta & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & G_{zx} \Delta
\end{pmatrix}
\]  

(2.4)

where

\[
\Delta = \frac{1 - v_{xy}v_{yz} - \nu_{yz}v_{zy} - \nu_{xx}v_{xz} - \nu_{xy}v_{yz}v_{xx} - \nu_{yx}v_{zy}v_{xx}}{E_x E_y E_z}
\]  

(2.5)

In the case of a transversely isotropic material with the axis of transverse isotropy along a given axis, the elastic deformation parameters in the plane orthogonal to the transverse isotropy direction do not differ (Ting, 1996). This means that the Young moduli, shear moduli and Poisson ratios in that plane are identical. For instance, for transverse isotropy along the \(z\) axis, this means that \(E_x = E_y\), \(G_{xz} = G_{yz}\), and \(\nu_{xz} = \nu_{yz}\). \(G_{xy}\) then depends on known parameters. The amount of unknown material parameters is therefore limited to five for a transversely isotropic material.

For a material with the maximum symmetry, i.e. an isotropic material, the material properties are the same in every direction. The elasticity matrix
Chapter 2. Finite Element Modeling of Elastic Membranes and Volumes

of an isotropic material then has the following symmetric form

\[
D = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \begin{pmatrix}
1 & \frac{\nu}{(1-\nu)} & \frac{\nu}{(1-\nu)} & 0 & 0 & 0 \\
\cdot & 1 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & 1 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \frac{1-2\nu}{2(1-\nu)} & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \frac{1-2\nu}{2(1-\nu)} \\
\end{pmatrix}
\tag{2.6}
\]

where Young’s modulus is the same in any direction: \(E = E_x = E_y = E_z\) and Poisson’s ratio too: \(\nu = \nu_{xy} = \nu_{xz} = \nu_{yz}\). There are no independent shear moduli, as the material parameters are the same in every direction. This is therefore reducing the total amount of material parameters to be determined to two.

Equation 2.1 is valid whether one is working with a surface or a volume. We model our active surfaces, which represent the boundaries of the objects in the image, as elastic membranes, and the surrounding and inner volumes as 3D volumetric elastic bodies.

Within a finite element discretization framework, an elastic body is approximated as an assembly of discrete finite elements interconnected at nodal points on the element boundaries. This means that the volumes to be modeled need to be meshed, i.e. divided into elements. Our volumetric and surface meshing algorithms will be described in Chapter 3. Figure 2.1a illustrates this discretization concept for a 2D object.

The continuous displacement field \(u\) within each element is approximated as a function of the displacement at the element’s nodal points \(u^{el}_i\) weighted by its shape functions \(N^{el}_i = N^{el}_i(\mathbf{x})\) (2.7).

\[
u = \sum_{i=1}^{N_{\text{nodes}}} N^{el}_i u^{el}_i
\tag{2.7}
\]

The elements we use are tetrahedra (number of nodes per element \(N_{\text{nodes}} = 4\)) for the volumes and triangles for the membranes (\(N_{\text{nodes}} = 3\)), with linear interpolation of the displacement field. Hence, the shape function of node
2.1 Mathematical Formulation

$i$ of element $el$ is defined as:

\[ N_{i}^{el} = K \left( a_{i}^{el} + b_{i}^{el}x + c_{i}^{el}y + d_{i}^{el}z \right) \]  

(2.8)

where \( K = \frac{1}{6V_{el}} \) for a tetrahedron, and \( K = \frac{1}{2S_{el}} \) for a triangle. The computation of \( V_{el}, S_{el} \) (volume, surface of $el$) and the interpolation coefficients is detailed by Zienkewicz and Taylor (1987), pages 91–92.

Figure 2.1 illustrates these concepts on a 2D object. The domain of the object is discretized using finite elements (triangles in Figure 2.1a), forming a mesh. The discretization of the domain allows us to solve for displacements only at the location of the discretization nodes of the domain. The displacement within each element is linked to the nodal displacements through its associated shape functions. Through such a discretization, and because the integral over the whole domain can be seen as the sum of the integrals over every element, it is possible to evaluate the equilibrium equations separately on every element, and to sum up the contribution of every triangle to which a vertex is connected to build a global equilibrium matrix system.

Figure 2.1: a) 2D Triangular FE mesh. b) Linear shape function of node $i$ of the triangular element.

For every node $i$ of each element $el$, we define the matrix $B_{i}^{el} = L_{i}N_{i}^{el}$ (where $L_{i}$ is matrix $L$ at node $i$, see Eq. 2.2). The function (see Eq. 2.1)
to be minimized on each element $el$ can thus be expressed as

$$E(u^el_1, \ldots, u^el_{N_{nodes}}) = \frac{1}{2} \int_\Omega \sum_{i=1}^{N_{nodes}} \sum_{j=1}^{N_{nodes}} u^el_i B^el_i^T D B^el_j u^el_j \, d\Omega$$

$$+ \int_\Omega \sum_{i=1}^{N_{nodes}} F N^el_i u^el_i \, d\Omega \quad (2.9)$$

We seek the minimum of this function by solving for

$$\frac{\partial E(u^el_1, \ldots, u^el_{N_{nodes}})}{\partial u^el_i} = 0 \quad ; \quad i = 1, \ldots, N_{nodes} \quad (2.10)$$

Equation 2.9 then becomes:

$$\int_\Omega \sum_{j=1}^{N_{nodes}} B^el_i^T D B^el_j u^el_j \, d\Omega = -\int_\Omega F N^el_i \, d\Omega \quad ; \quad i = 1, \ldots, N_{nodes} \quad (2.11)$$

This last expression can be written as a matrix system for each finite element:

$$K^{el} u^{el} = -F^{el} \quad (2.12)$$

Matrices $K^{el}$ and vector $F^{el}$ are defined as follows.

$$\begin{cases} K^{el}_{i,j} = \int_\Omega B^{el_i T} D B^{el_j} \, d\Omega \\ F^el_i = \int_\Omega F N^el_i \, d\Omega \end{cases} \quad (2.13)$$

where every element $i, j$ refers to pairs of nodes of the element $el$ ($i$ and $j$ range from 1 to 4 for a tetrahedron – 1 to 3 for a triangle). $K^{el}_{i,j}$ is a 3 by 3 matrix, and $F^{el}_i$ is a 3 by 1 vector. The 12 by 12 (9 by 9 for a triangle) matrix $K^{el}$, and the vector $F^{el}$ are computed for each element. The coefficients $i, j$ of the local matrices (corresponding to a pair of nodes $i, j$ within the local element) are summed up at the locations $g(i), g(j)$ in the global matrix (where $g(i)$ represents the number of the element’s node in the entire mesh). The assembly of the local matrices then leads to a global system

$$K u = -F \quad (2.14)$$

the solution of which will provide us with the deformation field corresponding to the global minimum of the total deformation energy.
We now have constitutive equations that model surfaces as elastic membranes and volumes as elastic bodies. Given externally applied forces $\mathbf{F}$ to a discretized body characterized by a rigidity matrix $\mathbf{K}$, solving Equation 2.14 provides us with the resulting displacements.

The resulting displacements can then be used to characterize the deformation the brain has undergone during the course of surgery using the stress and strain tensors. Using shape functions, stress and strain tensors can be derived from the displacement field at every nodal point $i$, given every tetrahedron $el$ of the FE model the node belongs to, and using the following relationships:

$$
\begin{align*}
\epsilon_i &= L_iN_i u_i \\
\sigma_i &= \sum_{el|i \in el} D \epsilon_i = \sum_{el|i \in el} D L_i N^{el}_i u_i
\end{align*}
$$

(2.15)

### 2.2 Implementation Considerations

The assembly of Equation 2.14 leads to a large matrix system, that needs to be solved. Typically, the brain meshes we use have around 25000 discretization nodes, leading to 75000 unknown displacements to solve for. Using classical direct solvers (such as those using Choleski factorization) leads to prohibitive computation complexity and memory storage requirements. The reason for this is that our matrix structure is sparse: the amount of elements in every row is limited by the connectivity of every node (i.e. the amount of incident nodes through an edge) in the FE mesh. Therefore, using iterative solvers or preconditioners (e.g. such as Jacobi or Conjugate Gradients) can benefit from the sparse matrix structure, both in terms of storage requirement and computational complexity. To solve our systems, we have chosen to use the Portable, Extensible Toolkit for Scientific computations (PETSc,Balay et al. (1998)) library. We have used the message passing interface (MPI) to parallelize the assembly of the matrix systems, and to optimize memory allocation of the different processors across the matrix system. We use a Generalized Minimal Residual (GM-Res) solver with block Jacobi preconditioning to solve our finite element equation systems.

When solving such large systems, the connectivity count of the nodes in the mesh will play an important role: the lower it is, the less elements
the matrix will have. Also, to optimize the parallelization of the assembly
and resolution of large systems, the connectivity count needs to be con-
stant throughout the mesh to avoid load balancing problems: if the nodes
managed by a processor have a higher connectivity than those of another
processor, it will have more computations to perform the system assembly.

Figure 2.2: Timing results for assembling, solving, and the sum of initial-
ization, assembling and solving times of 77511 equations simulating the FE
elastic deformation of a brain on the Deepflow cluster.

Figure 2.2 shows the speed-up we obtain when solving the equations sim-
ulating FE elastic brain deformation on the Deepflow cluster of Alpha
EV-56 workstations connected with fast ethernet (complete description at
http://deepflow.mema.ucl.ac.be). The results of this experiment show that
a simulation can be carried out in near real-time (i.e. less than ten seconds)
on a cluster for a typical brain model (Warfield et al., 2000a).
2.3 Illustration of Volume Deformation

In this experiment, we have simulated the compression of an isotropic elastic cube (with two material parameters $E$ and $\nu$ - see Eq. 2.6) fixed on the ground, whose bottom face has been constrained not to move. In Figure 2.3, the effect of varying Poisson coefficients $\nu$ is analysed with a constant Young modulus $E = 1 \, kPa$. A constant downward force of 1000 $N$ (Newton) has been applied to all the nodes of the top face of the cube, perpendicular to the latter face.

Figure 2.3: Deformation of an isotropic elastic cube ($E = 1 \, kPa, F = 1000 \, N$ ; (a,b) $\nu = 0$ ; (c,d) $\nu = 0.4$). (a,c) View of the deformed outer surface. (b,d) View of orthogonal cuts through deformed volume. For $\nu = 0$, the cube is linearly compressed and roughly retains its shape. For $\nu = 0.4$, the nearly incompressible material causes large shape changes and lateral expansion of the cube toward the bottom. Color-coding corresponds to the intensity of the deformation in $mm$. The pink wireframe is that of the original cube.
Figure 2.3a shows the deformed cube for $\nu = 0$, with orthogonal cuts through the volume in Figure 2.3b. Figure 2.3c,d illustrates the same, but with $\nu = 0.4$. The main result one can observe on this example is that, as expected, a compressible material (i.e. $\nu = 0$) causes linear compression of the cube in the direction of the applied forces (downwards in this example). This is illustrated by the fact that the deformed cube is still contained within the wireframe of the original cube (in pink in Fig. 2.3). On the other hand, shifting to less compressible materials (i.e. $\nu > 0$) causes a lateral expansion in the directions perpendicular to that of the applied force (horizontal expansion in this example). This can also be visualized in Figure 2.3 (subfigures b and d) where the deformed cube expands out of the wireframe of its initial shape. Getting very close to incompressibility ($\nu = 0.5$), causes numerical instabilities (due to the expression of the Elasticity matrix $\mathbf{D}$ - see Equation 2.6 or 2.4 - when solving for Equation 2.14).
In this chapter, we discuss the issue of discretizing bodies on which one needs to solve equations such as those described in Chapter 2. A brief review of surface and volumetric mesh generation is presented, followed by the description of a new approach we designed for generating meshes of complex domains (e.g. anatomical structures) from 3D images.

3.1 Motivation

In the developments we presented in Chapter 2, it is assumed that one has discretized the domain on which the equations need to be solved. The data available, medical images, is represented by an array of a finite number of image samples, or voxels. These can be used as discretizing elements of a finite element (FE) model. However, to limit computational complexity, it is desirable to work with less elements, suggesting that one may use elements covering several image samples. For computational ease and because they yield better representations of the domains we work upon, triangular elements are chosen to represent surfaces and tetrahedral elements for volumes, rather than directly working with the image samples.

There basically are 2 major kinds of approaches to surface and volume meshing: those based on the Delaunay criterion (Delaunay, 1934), and those based upon iso-voluming or iso-surfacing. There are also techniques known as contour linking techniques that extract parallel contours, and try to assemble them afterwards to reconstruct surfaces or volumes. We will
not consider such techniques, because they are not being used very much anymore. The interested reader can refer to Meyers et al. (1992) for surface reconstruction, and Geiger (1993) for volumetric reconstruction as examples.

The Delaunay criterion, also called the empty sphere property, says that any node of the mesh must not be contained within the circumsphere of any tetrahedra (triangles) in the mesh (Delaunay, 1934). This criterion has been exploited very intensively for triangular surface and volumetric tetrahedral mesh generation by Georges (1996) among others (see Owen (1998) for a complete review).

Iso-voluming and iso-surfacing algorithms proceed by clipping the elements of an initial regular subdivision of the domain to be meshed. The clipping of the elements is done if the element lies across a threshold (so-called iso-intensity value) crossing of underlying scalar content (e.g. gray-values of an image).

3.2 Surface Mesh Generation

3.2.1 Review of Existing Methods

This section reviews existing methods for generating triangular surface meshes.

Delaunay Based Algorithms

The Delaunay criterion can be used for generating 3D triangular surface meshes. This then requires an input set of points or a geometry from which the mesh is to be computed. A typical example of surface geometry are NURBS\footnote{NURBS: Non-Uniform Rational B-Splines are parametric functions which can represent any type of curves or surfaces.} that have been generated by some sort of CAD package. In the case of NURBS, the surface can be meshed in the parameter space using the Delaunay criterion, but this does not guarantee the element shapes back to x,y,z world space. Direct 3D surface mesh generation algorithms do not care about the parameterization and work directly on the input points. Because the only information we have available are the image samples, such techniques are not very well suited for our purpose.
Marching Cubes Algorithm

In the computer vision and graphics communities, the surface mesh generation algorithm that one usually refers to is the Marching Cubes algorithm, which has been developed by Lorensen and Cline (1987).

The principle of the algorithm consists of setting a threshold specifying the value above which the scalar content of the image is considered as being part of the object of which one desires to extract the surface.

For every voxel of the initial image, the gray-value at the vertex corners of the voxel cube is checked to see if it is above or below the requested threshold, called the iso-intensity value. Cubes having all its vertices above or below the iso-intensity will produce no facets to add to the cube. If a cube has a vertex above the iso-intensity and one below, this implies that the surface intersects the edge between those two vertices, and produces a configuration of one to four boundary facets. The facet configuration is looked up in a case table containing the $2^8 = 256$ possible edge intersection configurations. If one considers the symmetry within the case table, it can be reduced to 15 basic configurations, which are depicted in Figure 3.1.

![Figure 3.1: Different basic configuration cases for marching cubes iso-surfacing.](image)

However, when reconstructing the surface, there are some ambiguities, e.g. for case 10 in Figure 3.1. In that case, it is difficult to know what the
correct facet configuration is: given the intersection with the edges one cannot tell whether the four vertices above iso-intensity are separated by a gap (such as depicted here) or not (this would mean that the edge intersections would not intersect with the diagonal joining the two upper corners above iso-intensity). This ambiguity can be resolved by ensuring consistency between neighboring cubes, or by other methods as proposed by Nielson and Hamann (1991).

**Marching Tetrahedra**

Another means to avoid the ambiguity problem is to use an initial subdivision of the domain from which one wants to extract the isosurface into tetrahedra instead of cubes as proposed by Guéziec and Hummel (1995). The tetrahedralization of the initial domain can be done in several ways (see section 3.4.1). Proceeding this way, the case table for every tetrahedron only contains $2^4 = 16$ elements, with 3 basic cases if discarding all symmetrical cases, as depicted in Figure 3.2. No ambiguities are possible when reconstructing the iso-surface.

![Figure 3.2: Different basic configuration cases for Marching Tetrahedra iso-surfacing.](image)

Again, facets are added to the surface mesh only if the tetrahedron one is marching through is lying across the iso-surface in the image.

### 3.3 Volumetric Mesh Generation

#### 3.3.1 Review of Existing Methods

This section reviews existing methods for generating tetrahedral meshes.
3.3 Volumetric Mesh Generation

Delaunay Tetrahedralization

For numerical simulations such as those found in computational fluid dynamics, mechanical engineering, and materials engineering, the domains to be meshed are often well defined, and have very regular shapes (e.g. paraboloids, tubular elements, thin plates, ...). For these problems, the common approach to mesh the domain on which the Finite Element Analysis is to be performed, is the Delaunay tetrahedralization method based on the Delaunay criterion citep delaunay:34.

Most meshing techniques using this criterion either require an initial surface representation of the object to be meshed, or a set of input nodes that are part of the output mesh. Surface based techniques tetrahedralize the object by inserting nodes into the mesh, and redefining tetrahedra locally. It is the method that is chosen for defining where to locate the interior nodes that distinguishes one Delaunay algorithm from another (Owen, 1998). The main drawback of these meshing techniques is that they often require the objects to be meshed to be convex. This can be circumvented by splitting non-convex objects into smaller convex objects, and enforcing boundary conditions at the seams between sub-objects. When only input nodes are specified, there is no direct means to specify the inside and outside of the object and if it is non-convex, the algorithm will yield a mesh contained in the convex envelope of the input data.

We tested several publicly available Delaunay-based packages (Geiger (1993) and others found in Owen (1998)) on brain structures. Most of them failed, or generated an unacceptable amount of very small tetrahedra. Commercial packages such as TETMESH (Georges, 1996) distributed by Simulog 2 have a much more stable behaviour, but are extremely expensive.

Delaunay based algorithms require an input set of points or sets of boundary facets from which the mesh is to be computed. This requires a pre-processing step to extract significant boundary node positions from the image data. This process may be computationally as complex as the meshing itself. Another drawback is that these algorithms often replace input nodes and change the topology of the initial surface from which the tetrahedral mesh is to be computed to satisfy the Delaunay criterion. This is not always a desired feature.

The advantage of Delaunay based algorithm is that they provide a mesh

\[ \text{http://www.simulog.fr} \]
with well-shaped elements, that have near optimal aspect ratios (this means that the triangular facets of every tetrahedron have about the same size, with angles close to \( \pi/3 \)).

**Octree Mesh Representation**

The octree technique divides the elements (quads, cubes, tetrahedra) containing the geometric model recursively until the desired resolution is reached. It must be noted that the octree technique does not match a predefined surface mesh. To ensure element sizes do not change too dramatically, the maximum difference in octree subdivision level between adjacent cubes can be limited. Special care needs to be taken in order to ensure the consistency of the mesh for elements that lie next to elements that have been subdivided at a higher level.

Staadt and Gross (1998) describe a method to generate an octree-like subdivision of an image into tetrahedra for doing level-of-detail volume visualization. The idea is to recursively subdivide an initial tetrahedralization of the bounding box of the image until an error limit computed by isosurfacing on the current tetrahedral mesh (using marching tetrahedra) is reached.

**Iso-voluming**

Authors from the computer graphics community have developed a set of other tools for generating tetrahedral meshes for volume visualization. The ideas are the same as those proposed for iso-surface mesh generation, but instead of only extracting boundary facets, one extracts volumetric elements and the initial volumetric elements one is marching through are also added if they are contained in the object to be meshed.

Nielson and Sung (1997) present such an algorithm for tetrahedrizing image volumes that is a generalization of the iso-surface commonly associated with the marching cubes algorithm. The algorithm subdivides the image into hexahedra (e.g. voxels), and performs volumetric iso-contouring element by element using a pre-computed table containing the basic volumetric decomposition cases. The advantages of such a method is that it is fast and generates a very regular mesh except for the boundary elements that can have degraded aspect ratios (this can slow down further finite element computations). The main trouble with such an approach is that the average
size of the generated elements is determined by the initial size of the hexahedra into which the image is initially divided. Only the elements that have been iso-contoured are divided into smaller elements.

### 3.3.2 Our Approach

We have generated a new tetrahedral mesh generator, specifically suited for meshing anatomical structures using 3D labeled images, that combines the ideas of volume tetrahedralization such as proposed by Nielson and Sung (1997) (iso-voluming) and recursive mesh subdivision (octree subdivision).

An initial multi-resolution, octree-like tetrahedral approximation of the volume to be meshed is first computed depending on the underlying image content. Next, an iso-volume tetrahedralization is computed on the initial multi-resolution tetrahedralization so as to accurately represent the boundary surfaces of the objects depicted in the image. Figure 3.3 shows the different steps involved in our multiresolution tetrahedralization algorithm.

![Image](image)

**Figure 3.3:** Block scheme of multi-resolution mesh generator.

The algorithm first subdivides the image into cubes of a given size. This size will determine the resolution of the coarsest tetrahedra in the resulting mesh. The cubes are then tetrahedralized. At locations where the mesh needs better resolution (i.e. smaller edges), the tetrahedra are further divided adaptively into smaller tetrahedra, yielding an octree-like mesh. This subdivision causes cracks for tetrahedra that are adjacent to subdivided
Chapter 3. Finite Element Mesh Generation

tetrahedra. We solve this problem by re-meshing the neighboring tetrahedra using a precomputed case table. The resulting mesh contains pyramids, as well as prisms, that are further tetrahedralized.

Finally, the image labels of each vertex of every tetrahedron of the mesh are checked in a marching tetrahedra fashion. If the tetrahedron lies across the boundary of two objects with a different label, it is subdivided along the edges on the image's boundary so as to have an exact representation of the boundary between the objects. The latter is done using a precomputed case table, similar to that of the marching tetrahedra. The node positions on edges lying across object boundaries are found by interpolating the voxel values along the edge. The node is placed where the voxel value matches the iso-intensity value. The resulting mesh contains prisms, that we further tetrahedralize.

As an alternative approach, we could have used a cubic octree, and tetrahedralized the remaining cubes and pyramids next. If a hexahedron is to be subdivided in an octree fashion, it gets subdivided into 8 cubes. Each cube gets subdivided into 5 tets, which then yields 40 tetrahedra. If one does the octree subdivision on the tetrahedra of the initial subdivision of the hexahedron, this also yields 40 tetrahedra. The main advantage of working with tetrahedra from the beginning on is a greater flexibility as to which tetrahedra (or even just which edges) one desires to subdivide. The only disadvantage of the latter method is that if every tetrahedron of the initial subdivision is split into 8 tetrahedra, the connectivity of the nodes that have been added on the initial edges will be much greater than that of the initial nodes (up to 14 connected nodes against 6 for nodes belonging to an initial tetrahedralization of hexahedra).

3.4 Description of our Algorithm

3.4.1 Consistent Tetrahedralization of Hexahedra

The algorithm first tetrahedralizes the whole domain (i.e. the 3D image) by subdividing it into cubes of a given size. This size can be adjusted to vary the initial resolution of the mesh. The cubes can be subdivided into tetrahedra in several ways, but one has to keep in mind that to ensure efficient FE matrix computations it is important to limit the mesh connectivity as much as possible.
Continuous Pattern of Twelve Equally-sized Tetrahedra

It is only possible to subdivide a hexahedron into either five or six tetrahedra without adding additional data points. By adding the hexahedron centroid as data point, we can produce a subdivision into 12 tetrahedra, where each face is still split by a single diagonal. From here, we can progressively add face centroids, splitting a face into four triangles to produce 14, 16, 18, 20, 22 or 24 tetrahedra.

Ideally, we would like to be able to subdivide a mesh using splittings into five or six tetrahedra. This avoids the large jump to twelve, but more importantly, avoids the difficulties of adding new data points to the mesh.

Alternating Pattern of two Subdivisions into Five Tetrahedra

When a hexahedron is subdivided into five tetrahedra, the diagonals of parallel faces are crossed, so an alternating pattern of two subdivisions is needed to ensure the consistency of the mesh. Figure 3.4 illustrates the two alternated subdivisions of a hexahedron into five tetrahedra. This solution yields the smallest amount of tetrahedra. The tetrahedra are also better shaped, and the mesh connectivity is optimal in this configuration.

Figure 3.4: Two possible subdivisions of a cube into five tetrahedra.

Continuous Pattern of Subdivision into Six Tetrahedra

It is possible to avoid the subdivision of a hexahedron into an alternating pattern by subdividing a hexahedron into six tetrahedra. In this case, the diagonals of parallel quadrilateral faces have the same orientation, and hence, there are no problems for interconnecting hexahedra subdivided into tetrahedra in a consistent way. Figure 3.5 illustrates the subdivision of a
cube into six tetrahedra. The vertices labeled with a black spot illustrate two compatible diagonals on parallel faces of the initial hexahedron. The hexahedron is separated into two prisms that are each divided into three tetrahedra.

Figure 3.5: Subdivision of a cube into six tetrahedra.

The main disadvantage of such a subdivision is that the resulting tetrahedra do not have a good aspect ratio, and all the nodes of the mesh do not have the same connectivity.

3.4.2 Construction of a Multi-Resolution Tetrahedralization

The principle consists in subdividing tetrahedra covering image boundary regions by splitting their edges and adding a vertex at their middle point. By doing this, one obtains a list of edges to be split. Given the required edge splittings for each tetrahedron, there are six edges, thus $2^6$ possible configurations. We have implemented a table depicting all the possible configurations and their corresponding consistent tetrahedralizations. There are ten basic configurations, the remaining cases are symmetrical to these basic configurations, as depicted in Figure 3.6.

3.4.3 Tetrahedralization of Prisms and Pyramids

The resulting mesh contains tetrahedra, but also pyramids and prisms, which have to be further tetrahedralized. The main problem is to ensure consistency between the diagonals of quadrilateral faces shared by two elements (prisms or pyramids). There are different possibilities for choosing the diagonal on a quadrilateral face (see Nielson and Sung (1997)). We have chosen to split the quadrilateral faces along the shortest diagonal so as to have better shaped tetrahedra.

The division of a pyramid into two tetrahedra is straightforward given the diagonal of the quadrilateral face. Figure 3.7 presents the different possi-
3.5 Results on Synthetic Images

In this experiment, a synthetic sphere with a radius of 19 voxels was created within a 3D image of 64x64x64 voxels. The algorithm is tested on the whole volume with different parameters. Figure 3.9 shows renderings of meshes obtained with the algorithm with a regular initial grid (cubes of size 9x9x9) before clipping (sub-figures a,b,c), and with an initial mul-

Figure 3.6: Different subdivision of a tetrahedron given edge splittings.

...bilities for tetrahedralizing a prism given its quadrilateral face diagonals. As pointed out by Albertelli and Crawfis (1997), it is not clear if a valid subdivision is possible without adding cell or face centroids. If there is no straight tetrahedralization possible for the prism (cases 1 or 8 in Figure 3.7), a vertex is inserted in the middle of the prism and it is further divided into eight tetrahedra.

3.4.4 Accurate Representation Boundary Surfaces

Once the initial multi-resolution tetrahedralization of the domain has been performed, we extract a tetrahedral mesh of the objects in the initial image, along with precise boundary surfaces. This is done in a way similar to that of generating isosurfaces with a Marching Tetrahedra algorithm. The image labels of all the nodes of the multi-resolution mesh are checked, and for every tetrahedron its configuration is looked up in a case table. There are four vertices thus $2^4 = 16$ different cases, which can be summarized in five basic cases, the other ones being symmetrical to the basic cases. Figure 3.8 depicts the five basic cases given the nodes' label configuration.

3.5 Results on Synthetic Images

In this experiment, a synthetic sphere with a radius of 19 voxels was created within a 3D image of 64x64x64 voxels. The algorithm is tested on the whole volume with different parameters. Figure 3.9 shows renderings of meshes obtained with the algorithm with a regular initial grid (cubes of size 9x9x9) before clipping (sub-figures a,b,c), and with an initial mul-
Figure 3.7: Different subdivisions of a prism given the quadrilateral faces’ diagonals.

Figure 3.8: Different subdivisions of a tetrahedron given the nodes’ label configuration.
3.5 Results on Synthetic Images

tiresolution subdivision (five successive splittings) (subfigures d,e,f). One can very well observe the subdivision in the neighborhood of the sphere boundary in Figure 3.9d,e,f.

![Figure 3.9: Tetrahedral meshes of an embedded sphere. (a,b,c) Initial subdivision of image into 9x9x9 cubes, followed by clipping of the sphere. The boundary sphere has limited resolution, but most elements have a good aspect ratio. (c,d,e) Multi-resolution subdivision of initial cube of size 63x63x63 (entire volume) until smallest edge size was 2, followed by clipping. The boundary surface is more precise, but the shape of several elements is degraded. (a,d) Cut through tetrahedral mesh overlayed on corresponding cut through image. (b,e) 3D renderings of wireframe of entire mesh. (c,f) 3D Surface renderings of boundary surface of the sphere.](image)

The normal mesh without multiresolution had 2695 tetrahedra, and 344 triangular boundary facets, while the multiresolution mesh had 19232 tetrahedra and 4272 triangular boundary facets. Note that in order to achieve the same accuracy at the boundary surface without multiresolution, the mesh would have had 148955 tetrahedra and 7856 triangular boundary facets. The use of an initial multiresolution representation of the mesh around the boundary of the sphere greatly reduces the amount of tetrahedra of the mesh.
For finite element computations, a multiresolution mesh such as the one shown on the bottom row of Figure 3.9 is not very optimal as the connectivity of the nodes of the mesh can greatly vary. For instance, the connectivity of the corner vertices is very high. Also, when performing multiple subdivisions, elements next to those getting subdivided end up with a bad aspect ratio. Using a finer initial subdivision and less subdivision edges yields better shaped elements, with a lower connectivity count of the nodes.

### 3.6 Mesh Quality Assessment

The quality of an element is crucial for further Finite Element analysis. A perfect element has triangular facets having the same size, with angles of $\pi/3$. Elements having degraded aspect ratios (e.g., one edge is much shorter than the others) have a lower quality. Poorly shaped or distorted elements can result in numerical difficulties during the solution process. For example, it has been shown that as element angles become too large, the discretization error in the finite element solution is increased and as angles become too small the condition number of the element matrix is increased (Freitag and Ollivier-Gooch, 1996). Thus, for meshes containing distorted elements, the numerical solution is more difficult to compute and the numerical approximation is less accurate. Most tetrahedral quality measures are based on geometric quality indicators (Parthasarathy et al., 1993; Berzins, 1999). One common example is the so-called aspect ratio, defined as

$$ A_\gamma = \left( \frac{\frac{1}{6} \sum_{i=1}^{6} l_i^2}{8.47867 V_{el}} \right)^{3/2} $$  \hspace{1cm} (3.1)$$

where $V_{el}$ is the volume of the tetrahedron, and $l_i \ (i = 1, ..., 6)$ are its edge lengths. The aspect ratio metric is normalized so that $A_\gamma = 1$ corresponds to an ideal element and $A_\gamma \to \infty$ as the element becomes increasingly distorted.

To assess the quality of the meshes our algorithm generates, and to improve the low quality elements, we have used the quality measure of Equation 3.1. For the first experiment presented in the previous section (sphere with constant subdivision before clipping), the average aspect ratio of the mesh was
3.7 Discussion

31.5, but the maximum aspect ratio was 8077. 85% of the elements had aspect ratios below 5, but among the 13% remaining tetrahedra, some are quite distorted. For the second experiment (sphere with multiresolution subdivision before clipping), the average aspect ratio of the mesh was 22.6 and the maximum aspect ratio was 16084. 77% of the elements had aspect ratios below 5, and some of the remaining 23% tetrahedra have extremely distorted elements. Note that the average aspect ratios without clipping the mesh at the boundaries were respectively 1.0006 (maximum 1.29) and 2.47 (maximum 28), which tells us that element quality needs to be improved mainly in the vicinity of the boundary surfaces.

To improve the shape of those elements that had distorted shapes, we have implemented a simple laplacian smoothing algorithm, and applied it in the second order neighborhood (i.e. up to two edges away) of all the tetrahedra having a degraded aspect ratio. Laplacian smoothing relocates grid points to the mean of their incident vertices to improve mesh quality without changing mesh topology. The laplacian smoothing is applied iteratively, by ensuring that the smoothing is not degrading other element’s shapes at every iteration, and without moving the boundary surface nodes. In both examples mentioned above, tetrahedral mesh quality improved significantly. For the first mesh, the percentage of poorly shaped elements went down to 6.29% and for the second one it went down to 11.75%. The average aspect ratios of the meshes improved to respectively 15.6 and 18.3. The maximum aspect ratios barely changed, indicating that the mesh smoothing failed to improve the worst quality elements.

3.7 Discussion

The meshes generated by the algorithm we have described here can be customized so as to produce tetrahedra having a size varying with the underlying image content. This can greatly reduce the amount of generated tetrahedra as compared to a method such as proposed by Nielson and Sung (1997).

However, as discussed above, the resulting elements do not always have optimal aspect ratios. One solution, within our algorithm, is to limit the amount of subdivisions by choosing a smaller initial tetrahedralization.

However, it is always desirable to post-process the mesh to improve the
shape of the degraded elements in the mesh. There are mainly two categories of mesh improvement: those that use some form of smoothing, and those doing clean-up (Owen, 1998). Smoothing adjusts node locations to improve element shape, but leaves connectivity unchanged. Clean-up refers to any process that changes the element connectivity. We investigated a very simple form of mesh smoothing (laplacian smoothing), and showed that it can effectively improve the overall mesh quality. However, in order to completely eliminate highly degraded elements from the mesh, we believe more sophisticated methods (e.g. Freitag and Ollivier-Gooch (1996)) need to be investigated.
Chapter 4

Deformable Surface Modeling

In this chapter, we describe how one can use equations modeling elastic membranes (such as presented in Chapter 2) to solve segmentation, shape recovery, and tracking problems in 3D images by the means of deformable surface models. A brief review of the theory of deformable surfaces is presented and followed by the description of our approach to deformable surface tracking. A few experiments on synthetic image data illustrate the behavior of the algorithm we implemented.

4.1 Description of our Algorithm

Deformable contours, also called active contours or snakes, were introduced by Kass et al. (1988), and have been increasingly used since then in the computer vision and medical image analysis communities (see McInerney and Terzopoulos (1996) for a review on medical applications and Audette et al. (2000) for a review on algorithmic types of deformable contours) for segmentation, registration and shape tracking or shape recovery.

An active contour is characterized by three parts:

- **internal forces**: these are the elasticity and bending moments describing the contour as a physical object (e.g. string). These forces are designed to hold the curve together, and locally smooth (first order terms), as well as keeping it from bending too much (second order terms).
• **external forces**: these forces describe how the active contour is attracted to the desired features of the image data.

• **iterative procedure**: the iterative procedure attempts to find the configuration that best matches both the internal and external forces.

Bending moments are not always a desired feature for mapping sharp folds, since they do not allow for sharp transitions in the surface geometry (Davatzikos and Prince, 1995). If the bending term is omitted, the active contour (surface) may be seen as an elastic string (membrane), having only first order terms in the partial derivative equation modeling its behavior.

The 2D active contour model has been extended to 3D surfaces by Cohen and Cohen (1993), who also proposed to discretize the resolution of the equations governing the behavior of the surfaces using finite elements. The temporal (iterative) variation of the surface can be discretized using finite differences, provided the time step $\tau$ is small enough. The surface is then deformed iteratively by applying image-derived forces $F^v_t$ (forces computed using the surface’s nodal positions $v$ at iteration $t$) to the active surface. Using the constitutive equation for elastic membranes for the active surface (see Eq. 2.14), this yields the following semi-implicit iterative equation ¹:

$$\frac{v^t - v^{t-1}}{\tau} + Kv^t = -F^{v_{t-1}}$$

which can be rewritten as:

$$(I + \tau K)v^t = v^{t-1} - \tau F^{v_{t-1}}$$

The external forces driving the elastic membrane towards the edges of the structure in the image are integrated over each element of the mesh and distributed over the nodes belonging to the element using its shape functions (see Eqn. 2.7). Classically, the image force $F$ is computed as a decreasing function of the gradient so as to be minimized at the edges of the image (Kass et al., 1988; Cohen and Cohen, 1993). A potential weakness of active surface methods is that for correct convergence, the surfaces need to be initialized very close to the edges of the object to be segmented. In Cohen and Cohen (1993), the authors proposed to use inflation or deflation forces (so-called balloon forces) to circumvent that problem and increase the capture range of the deformable surface. Xu and Prince (1998) compute the force field driving the curve separately on the segmented target

¹Superscript $t$ refers to the current iteration.
image by solving a separate second order differential equation coming from electromagnetics. The main disadvantage of such an approach is that it is computationally prohibitive in 3D.

To increase the robustness and the convergence rate of the surface deformation, we compute our forces as a gradient descent on a distance map of the edges in the target image. The distance map is computed very efficiently using a fast distance transformation algorithm (Cuisenaire and Macq, 1999). To prevent the surface from sticking on a wrong edge, or to prevent two sides of a thin surface from sticking together on the same edge, we have included the expected gradient sign of the edges of the structure to be segmented in the force expression (Ferrant et al., 1999a).

This yields the following relation for the external force:

$$ F(x) = S_{min} G_{exp} \nabla \left( D(I(x)) \right) $$  \hspace{1cm} (4.3)

where $D(I(x))$ represents the distance transformation of the target image at point $x$. $S_{min}$ is chosen so that the gradient points towards a point with a smaller distance value:

$$ S_{min} = \begin{cases} +1 & \text{if } D(I(x)) > D(I(x + \nabla D(I(x)))) \\ -1 & \text{if } D(I(x)) < D(I(x + \nabla D(I(x)))) \end{cases} $$

$G_{exp}$ is the contribution of the expected gradient sign on the labeled image:

$$ G_{exp} = \begin{cases} +1 & \text{if } k n \nabla I(x) > 0 \\ -1 & \text{if } k n \nabla I(x) < 0 \end{cases} $$

$k = +1$ if the region to be matched has a higher label value than the surrounding tissues; $k = -1$ if the region to be matched has a lower label value than the surrounding tissues. This means that if a surface whose normal points in the opposite direction to that of the expected image gradient, it will not be attracted towards that edge.

### 4.2 Initialization of the Deformable Surface

As mentioned in the previous section, a potential problem with active surfaces is that they often need to be initialized close enough to the object onto which they need to be matched. Often, inflation/deflation forces and
prior gradient information do not suffice to obtain correct convergence of the active surface.

When one has prior information about the surface to be matched (e.g. in the case of atlas matching, or in the case of tracking), an initial global repositioning of the surface to be matched can be very useful to account for global shape changes such as rescaling and rotation. To do so, we use the algorithm proposed by Cuisenaire et al. (1996); Cuisenaire (1999). The idea of matching edge points from two images using a Distance Transform (a Chamfer map) was proposed by Borgefors (1988) and Jiang et al. (1992a) and also used for surface matching by Jiang et al. (1992b).

Similarly to Mangin et al. (1994), we define the registration criterion as a distance measure between the extracted discrete surface $S_{mob}$ and the equivalent reference object in the atlas $S_{ref}$.

$$D(S_{mob}, S_{ref}) = \left( \frac{\sum_{x \in S_{mob}} d_{S_{ref}}^2(x)}{\#(S_{mob})} \right)^{\frac{1}{2}}$$  \hspace{1cm} (4.4)

with

$$d_{S_{ref}}(x) = \min\{d(x, y) \mid y \in S_{ref}\}$$  \hspace{1cm} (4.5)

This distance measure can be efficiently computed by precomputing the distance $d_{S_{ref}}$ from any pixel $x$ to the reference surface $S_{ref}$ using the Distance Transform algorithm described by Cuisenaire and Macq (1999); Cuisenaire (1999), which is mainly an efficient implementation of the Euclidean Distance Transform proposed by Danielsson (1980). True Euclidean distance is indeed required here since its commonly used Chamfer approximations (such as used by Borgefors (1988) and Jiang et al. (1992a)) fail to provide a good estimate of the gradient of the distance. The global transformation is then considered as the linear combination of 30 de-correlated basic transformations whose coefficients are optimized by a steepest gradient minimization of the criterion in the 30-dimensional coefficient space.

Such a transform provides us with a good initialization for running an active surface algorithm next that can then account for local shape changes of the surface.
4.3 Results on Synthetic Images

4.3.1 Sphere to Cube Experiment

In this experiment, a sphere surface with a radius of 19 centered about coordinate (32,32,32) is matched onto an image of a cube with edges of 15 whose center is at coordinate (37,37,37). Figure 4.1 illustrates the deformation of the surface at the different stages.

![Figure 4.1: a,b,c) 3D Surface renderings of initial, affine transformed and deformed surfaces. d,e,f) Cuts through initial, affine transformed and deformed surfaces overlayed on corresponding cut through target image. The affine transform captures global differences (translation, scaling), while the active surface deformation accounts for local shape differences.](image)

The initial surface is shown in the first column of Figure 4.1. Subfigure 4.1a) is a 3D rendering of the initial surface, 4.1b) illustrates a cut through the surface overlayed on a corresponding cut through the target image. The middle column of Figure 4.1 (subfigures c,d) depicts the same surface after affine transformation as initialization. The right column of Figure 4.1 (subfigures c,d) illustrates the surface after deformation with the active surface algorithm. Note that the affine transformation has translated the sphere surface, and has schrunk it a little bit too so it is better initialized around the cube. After deformable surface deformation, the edges of the
cube are rounded partly because of the limited resolution of the surface, but also because the elasticity of the surface prevents it from bending too much.

4.3.2 Sphere to Paraboloid Matching

In this experiment, the active surface algorithm is used to match a sphere onto a paraboloid whose equation was

\[
\left( \frac{x - 32}{12} \right)^2 + \left( \frac{y - 32}{12} \right)^2 - \frac{z - 20}{15} < 0.8 \quad (4.6)
\]

with \( z < 50 \). The paraboloid is embedded into a 64x64x64 image. Figure 4.2 illustrates the application of our algorithm on the data. Again, the edges of the target surface are rounded because of the elasticity of the surface.

![Figure 4.2: a,b) Cuts through the initial and deformed surfaces overlayed on corresponding cut through target image. c) 3D Surface rendering of target surface after active surface deformation.](image-url)
Chapter 5

Volumetric Elastic Image Matching

In some cases, it can be useful to deform volumes rather than just surfaces, as surfaces are often used to represent volumetric objects. Thus instead of matching a surface onto a target image, one could deform an initial image onto a target image. When this is done by imposing regularity constraints, such an approach also has the advantage of maintaining the spatial relationship between the objects depicted in the initial image.

In this chapter, we present an algorithm for doing elastic image matching using Finite Element discretization. The idea is to modify the constitutive equation of volumetric bodies (as presented in Chapter 2) to incorporate an image similarity constraint into the expression of the potential energy of an elastic body submitted to external forces (Equation 2.1). The elastic potential energy then serves as a physics-based regularity constraint to the image similarity term.

5.1 Motivation

During the last decade, physically realistic models for surgical planning and image registration have gained increased attention in the medical imaging community. The reason for this is that purely image-based statistical methods do not take into account the physical properties of the objects depicted in the image and often cannot predict any changes in the image. This is especially true when one is dealing with medical image data. Different imaged objects have very different properties and react in a way defined by
their material characteristics (e.g. bone and soft tissue have a very different behavior when submitted to equivalent stresses). Therefore, we believe that using a model incorporating the object’s physical characteristics can improve the accuracy of deformable registration significantly.

Previous work for recovering image deformation is mainly based on local image structure (Dengler and Schmidt, 1988; Bauchemin and Barron, 1995). These methods compute a deformation field between images simultaneously minimizing a local similarity measure and satisfying some kind of arbitrarily chosen smoothness constraint. They are often referred to as optical flow (OF) methods. Later, the image registration community proposed physical deformation models to constrain the deformation field using elastic (Bajcsy and Kovacic, 1989; Davatzikos, 1997) or even viscous fluid deformation models (Bro-Nielsen and Gramkow, 1996; Christensen et al., 1997).

In this chapter, we present a new finite element (FE) based elastic image matching algorithm. We propose an integrated approach that implicitly computes the forces applied to the 3D model by constraining the deformation field to satisfy both the elasticity model and the local image similarity criterion. This is achieved by embedding an image similarity constraint on the deformation field into the minimization scheme that leads to the constitutive equations of the deformation model. The equations are discretized using the finite element method.

5.2 Mathematical Formulation

We formulate the elastic matching of two images as an energy minimization procedure, where the energy comprises a term modeling the physical behavior of the object to be deformed and another term driving the model so as to match both images. The matching criterion between both images is modeled as the minimization of the sum of the squared differences between both images.

In the expression of equilibrium equations presented in Chapter 2 (see Equation 2.14 the external forces $F$ could be computed as a classical optical flow field between the images to be matched, providing us with a semi-implicit method where the optical flow field would be an initial estimate of the deformation field being regularized by the elastic model. The estimates can
then be iteratively refined until an equilibrium is reached.

To avoid the separate computation of the forces $\mathbf{F}$, the elastic deformation, and the matching criterion, we propose to directly compute a deformation field that readily satisfies both the elasticity constraint and a local image similarity constraint between the images to be matched ($I_1$ and $I_2$). Hence, the total deformation energy to be minimized (see Equation 2.1) is expressed as

$$E = \frac{1}{2} \int_{\Omega} \sigma^T \epsilon \, d\Omega + \frac{1}{2} \int_{\Omega} (I_1(x + \mathbf{u}) - I_2(x))^2 \, d\Omega$$  \hspace{1cm} (5.1)

Assuming that the deformation field is small and the variation of $I_1$ smooth, the first order Taylor expansion of $I_1(x + \mathbf{u})$ can be expressed as

$$I_1(x + \mathbf{u}) \cong I_1(x) + (\nabla I_1(x)) \mathbf{u}$$  \hspace{1cm} (5.2)

Using the material’s constitutive Equation 2.3 and Equation 5.2, Equation 5.1 becomes (the dependencies to $\mathbf{x}$ are omitted in further developments to clarify the equations):

$$E = \frac{1}{2} \int_{\Omega} \epsilon^T D \epsilon \, d\Omega + \frac{1}{2} \int_{\Omega} (I_1 - I_2)^2 - 2(I_1 - I_2) \nabla I_1 \mathbf{u} + \mathbf{u}^T \nabla I_1^T \nabla I_1 \mathbf{u} \, d\Omega$$  \hspace{1cm} (5.3)

As in Chapter 2, we discretize these equations on a domain described by tetrahedral finite elements (amount of nodes $N_{\text{nodes}} = 4$), with linear interpolation shape functions $N_i^{\text{el}}$ linking the continuous displacement to the displacement of the discretization nodes. Similarly to what we did in Chapter 2 (Equation 2.9), for every element $\text{el}$, we define the matrices $\mathbf{B}_i^{\text{el}} = \mathbf{L}_i N_i^{\text{el}}$. The function to be minimized on each element $\text{el}$ can thus be expressed as

$$E(\mathbf{u}_1^{\text{el}}, \ldots, \mathbf{u}_4^{\text{el}}) = \frac{1}{2} \int_{\Omega} \sum_{i=1}^{4} \sum_{j=1}^{4} \mathbf{u}_i^{\text{el}T} \mathbf{B}_i^{\text{el}} D \mathbf{B}_j^{\text{el}} \mathbf{u}_j^{\text{el}} \, d\Omega$$

$$+ \int_{\Omega} \sum_{i=1}^{4} (I_1 - I_2)^2 - 2(I_1 - I_2) \nabla I_1 N_i^{\text{el}} \mathbf{u}_i^{\text{el}} \, d\Omega$$

$$+ \int_{\Omega} \sum_{i=1}^{4} \sum_{j=1}^{4} \mathbf{u}_i^{\text{el}T} N_i^{\text{el}} \nabla I_1^T \nabla I_1 N_j^{\text{el}} \mathbf{u}_j^{\text{el}} \, d\Omega$$  \hspace{1cm} (5.4)
Again, we seek the minimum of this function by solving for
\[ \frac{\partial E(u^{el}_1, \ldots, u^{el}_4)}{\partial u^{el}_i} = 0 \; ; \; i = 1, \ldots, 4 \] (5.5)

Equation (5.4) then becomes \((i = 1, \ldots, 4)\)
\[ \int_\Omega \sum_{j=1}^4 \left( B^{el}_{i,j} T D B^{el}_{i,j} + N^{el}_i \nabla I_1 T \nabla N^{el}_j \right) u^{el}_j d\Omega = \int_\Omega (I_1 - I_2) \nabla I_1 N^{el}_i d\Omega \] (5.6)
This last expression can be written as a matrix system for each finite element :
\[ (K^{el} + G^{el}) u^{el} = F^{el} \] (5.7)
Matrices \(K^{el}, G^{el}\) and \(F^{el}\) are defined as follows :
\[
\begin{align*}
K^{el}_{i,j} &= \int_\Omega B^{el}_{i,j} T D B^{el}_{i,j} d\Omega \\
G^{el}_{i,j} &= \int_\Omega N^{el}_i \nabla I_1 T \nabla N^{el}_j d\Omega \\
F^{el}_i &= \int_\Omega (I_1 - I_2) \nabla I_1 N^{el}_i d\Omega
\end{align*}
\] (5.8)
where every element \(i, j\) refers to pairs of nodes of the element \(el\) \((i \text{ and } j \text{ range from } 1 \text{ to } 4)\). \(K^{el}_{i,j}\) and \(G^{el}_{i,j}\) are 3 by 3 matrices, \(F^{el}_i\) is a 3 by 1 vector. The 12 by 12 matrices \(K^{el}\) and \(G^{el}\), and the vector \(F^{el}\) are computed for each element. Again, similarly to what we explained in Chapter 2, the local matrices and vector of each element are then assembled in a global system
\[ (K + G) u = -F \] (5.9)
the solution of which will provide us with the deformation field corresponding to the global minimum of the total deformation energy. The matrix \(K\) is the rigidity matrix which is similar to the one of Equation 2.14 in Chapter 2.

5.3 Synthetic Experiments

Experiments with synthetic images were carried out to verify the plausibility of our model and to show the advantage, for medical imaging applications, of our matching using an elastic model instead of just considering local image structure with a smoothness constraint (Optical Flow methods).
5.3 Synthetic Experiments

5.3.1 Growing Sphere

We have applied the algorithm to a synthetic sequence of two spheres (gray regions) centered at the same location with a radius of 15 and 17 pixels. In Figure 5.1, one can observe that the deformation field yielded by OF is located only at the voxels where the difference between both images is non-zero, while the FE elastic deformation algorithm propagates the deformation all along the surrounding elastic body (which in this experiment was stiffer and had a larger Young modulus than the sphere itself).

![Figure 5.1: Growing sphere. a) and b): close-ups of 2D cuts through 3D image with a) classical OF, and b) FE matching deformation fields overlayed, c) and 3D orthogonal cuts through the FE mesh with intensity coding of the displacement field. The displacement field is mainly located at the boundaries of the sphere and is propagated through the surrounding elastic medium, whereas in the OF setting it is only located at those locations in the image where the images differ.](image)

5.3.2 Matching a Sphere onto a Cube

In this experiment we try to match a sphere onto a cube. The sphere was centered a few voxels above the cube. In order to achieve decent results, the algorithm had to be applied iteratively. Figure 5.2 shows the obtained deformation field overlayed on the sum image between the target and the deformed initial image at the different iterations of the algorithm.

One can very well observe the fact that the sphere starts by a downwards deformation and then progressively deforms towards the edges of the cube until its shape matches that of the cube. This deformation can also be observed very clearly when looking at a cut through the deformed mesh such as illustrated in Figure 5.3. One can very well see the sphere getting
Figure 5.2: Sphere to cube experiment. 2D deformation field overlayed on cut through target and deformed images at 4 successive iterations of the matching process. The resulting match illustrates the accuracy of the match and the distribution of the deformation field over the volume. translated down and attracted to the corners of the cube.

Figure 5.3: Sphere to cube experiment. Cut through deformed mesh overlayed on corresponding cut through target image. This illustrates how the deformation of the boundary of the sphere elastically stretches and compressed the volume into which the sphere is embedded.

5.4 Discussion

Compared to classical optical flow (OF) methods, there are several advantages to use an approach such as the one we have presented.

- The first advantage comes from the use of a FE formulation: one can assign different regularization parameters to different parts of the initial image causing some parts of the image to deform rigidly
and some others softly for instance. This is straightforward to do within a finite element framework.

- Second, posterior use of the initial and deformed FE models of the image can provide us with physical properties (such as stress measures) of objects deformed from one image to the next. This then assumes that the obtained deformation results from an actual physical deformation.

- Finally, one may consider a multiresolution approach by selectively refining an initial coarse mesh in the regions where more precision is needed without having to build a multiresolution pyramid of the entire input image.

The main disadvantage of such a method is that the deformations computed do not satisfy the actual constitutive equations of elasticity such as expressed by Equation 2.1. Even though one may consider characterizing the stress tensors of a physical deformation that would have yielded the same deformation, such an expression may not be considered as a long term solution towards physics-based simulation for instance. Another problem is that the expression used as image similarity criterion does not allow for multi-modality image matching. Also, such a method is sensitive to intensity inhomogeneity and varying contrast in both images. Approaches using a Mutual Information (MI) similarity criterion could for instance be used. Several solutions have been proposed (e.g. Gaens et al. (1998); Hata (1998); Maintz et al. (1998); Rueckert et al. (1999); Thiran and Butz (2000)) for doing non-rigid MI based matching. Other approaches using ridgeness (e.g. Thirion (1995)) and correlation (e.g. Collins et al. (1994)) have also been proposed, and could be incorporated into our elastic matching scheme.
Chapter 6

Using Deformable Surface Models with Volumetric Models

In this chapter, we present an algorithm for doing physics-based registration of images using boundary deformation of key objects to infer a volumetric deformation field. We use the algorithm presented in Chapter 4 to track the deformation of boundary surfaces of key objects in the images and we use that deformation as input to a volumetric FE elastic model (such as presented in Chapter 2). Such an approach yields a deformation field satisfying the constitutive equations of the body, and can be used to characterize the deformation the body has undergone from one image to the next. We illustrate these concepts on a couple of synthetic image sequences.

6.1 Motivation

Often, it is possible to track boundaries of objects in images using deformable surface models, but embedded objects do not always exhibit visible boundaries, and are therefore not easily trackable or segmentable. Also, when using volumetric registration methods such as that described in Chapter 5, the matching does not represent an actual physical deformation, since the energy functional that is minimized represents a weighted sum of the elastic potential energy of the body and of the squared difference between the images to be matched. Therefore, it is interesting to explore image registration applications with a physically meaningful and realistic deformation analysis. Also, such approaches are better suited if the long term
idea is to extend image registration to simulation.

Several authors have suggested to use volumetric deformable models in conjunction with surface based models to deform an image onto another (Thompson and Toga, 1996; Davatzikos, 1997), but their motivation was mainly to find an interpolation function to derive a volumetric deformation field from surface deformations.

We propose to use a finite element based volumetric biomechanical model driven by surface-based deformations to determine and characterize a volumetric biomechanical deformation field between images to be matched.

### 6.2 Description of our Method

In our model, boundary surfaces are embedded into a volumetric FE mesh model (such as presented in Chapter 3). We track boundaries in the images using an active surface model (see Chapter 4), and we use the deformation of the surface model to deform the volume the surface(s) is (are) embedded in.

The volume is modeled as an elastic body (see Chapter 2), and the embedded surfaces are deformed independently using an active surface algorithm. Once the embedded surface(s) has (have) been deformed, the initial volume is deformed by applying forces producing the same displacement as that obtained at the boundary nodes.

Let $\bar{u}$ be the vector representing the displacement to be imposed at the boundary nodes. The elements of the rows of the rigidity matrix $K$ corresponding to the nodes for which a displacement is to be imposed need to be set to zero, and the diagonal elements of these rows to one (Zienkiewicz and Taylor, 1987). The force vector $F$ is then set to be equal to the displacements vector for the boundary nodes: $F = \bar{u}$ (Zienkiewicz and Taylor, 1987). This way, solving Eq. 2.14 for the unknown displacements will produce a deformation field over the entire FE mesh model that matches the prescribed displacements at the boundary surfaces.

Figure 6.1 presents the flow diagram of our algorithm for matching one image onto another one using the idea of combining active surfaces and volumetric models. First step is to segment the key objects out of the ini-
6.3 Results on Synthetic Image Sequences

6.3.1 Embedded Translated Cube Experiment

In this experiment, the aim is to match a hard isotropic elastic cube (edges have a length of 15 $mm$, $E = 10kPa, \nu = 0$) embedded in a larger soft isotropic elastic cube (edges have a length of 64 $mm$, $E = 1kPa, \nu = 0.4$) onto a target cube of the same size, but whose smaller cube has been translated by a vector of (5mm,5mm,5mm). The boundary surface of the larger cube is assumed not to move, that of the smaller cube is translating within
Chapter 6. Using Deformable Surface Models with Volumetric Models

the larger one. Figure 6.2a represents a cut through the initial tetrahedral mesh overlayed on the corresponding cut through the image representing the first cube. Figure 6.2b,c represent the boundary surface of the smaller cube overlayed on the target image, and after surface matching. In this experiment, we only used the distance transform based global transformation algorithm to correct for the translation. In this case, the active surface algorithm alone would not have been able to recover such a large translation.

The boundary displacement of the smaller cube is then used as input to the volumetric FE model, which is then deformed. Figure 6.2d shows the same cut as on sub-figure (a) through the deformed mesh overlayed on the corresponding cut through the target image.

![Figure 6.2: Translated cube experiment. a) Cut through initial FE mesh overlayed on corresponding cut through image the mesh was generated from. b) Cut through initial boundary surface overlayed on corresponding cut through target image. c) Cut through deformed initial boundary surface (after affine transformation) overlayed on corresponding cut through target image. d) Cut through deformed FE mesh overlayed on corresponding cut through target image. This illustrates the rigid translation of the cube, pulling tissue of the upper left corner and compressing the soft tissue in the lower right corner.](image)

Figure 6.3a shows orthogonal cuts through the deformed mesh with color-coding of the intensity of the deformation field with the actual 3D deformation field. Figure 6.3b shows the 2D deformation field interpolated back onto the image grid and overlayed on corresponding cut through the initial image.

6.3.2 Embedded Sphere to Ellipsoid Experiment

We have tested the algorithm on a sequence of two 3D images of an elastic sphere being squeezed in a given direction. The object is surrounded by
6.3 Results on Synthetic Image Sequences

Figure 6.3: Translated cube experiment. a) 2D deformation field overlayed on cut through initial image. b) 3D rendering of cuts through deformed FE mesh with arrows representing the actual 3D deformation field. Color-coding corresponds to the intensity of the deformation. The deformation field is translational within the hard cube, and has a rotational component in the soft part towards the edges of the image boundaries.

Another soft elastic object.

The original active surface extracted from the volumetric tetrahedral mesh is shown in Figure 6.4. Note that for this experiment, the initial tetrahedralization from which the mesh was computed was not multi-resolution, it had constant tetrahedral sizes before the volumetric marching tetrahedra contouring was applied. When running the active surface algorithm, the surface readily converges to the boundary of the ellipsoid in the target image.

Figure 6.5 shows 3D renderings of cuts through the mesh associated to the initial image and of the mesh after deformation, while Figure 6.6 shows cuts through the original (a) and deformed mesh (b) and the deformation field (c) interpolated back onto the image grid (downsampled for clarity) overlayed on a cut through the target volume. One can clearly observe the physical squeezing of the sphere onto the ellipsoid, also deforming the surrounding elastic medium.
Figure 6.4: a) Slice 30 of the target image with a cut through the initial surface of the object overlayed. b) 3D surface rendering of the initial surface. c) The same slice with a cut through the deformed surface. d) 3D surface rendering of the deformed surface.

Figure 6.5: a) Orthogonal cuts through the initial volumetric mesh with the sphere extracted and b) the same with deformed mesh. These visualizations illustrate how the squeezing of the sphere onto the ellipsoid affects the surrounding elastic medium.
6.4 Discussion

There are multiple advantages in using such an approach:

- **Multi-modality**: compared to the method presented in Chapter 5, this algorithm does not need the images to be acquired with the same imaging modality because of the image similarity criterion. Hence, even with similar imaging modalities, the algorithm will be less sensitive to intensity inhomogeneity and contrast. The only primary requirement is that the key objects need to be trackable by the active surface algorithm.

- **Physically meaningful and interpretable deformation**: unlike the equations presented in Chapter 5, the constitutive equations of elastic bodies such as presented in Chapter 2 are respected and can thus be used to characterize the deformation.

- **A step towards simulation**: such an approach can be seen as a simulation tool with minimal external input data. The external input data in this context would be the imaging that serves to update the model according to the changes the image sequence depicts. This approach will be detailed in the context of intraoperative brain deformation tracking and analysis in Chapter 9.

The main disadvantage of the method presented is that the deformation field the active surface algorithm yields is supposed to be the physical deformation of the boundary of the object the surface tracks. This is of
of course not guaranteed, but if one chooses the active surface to deform smoothly, the result probably does not differ too much from the actual deformation. Also, the physics based model we currently use is limited to linear elasticity. Therefore, if it is used for physical characterization of deformation, the bodies need to be elastic bodies or have a quasi-elastic behavior (i.e. small strains) on what the images depict. Nevertheless, this assumption is often verified in the case of soft tissue deformation.
Part II

Applications
Chapter 7

Volumetric Elastic Image Matching : Results on Medical Data

In this chapter, we report on preliminary results obtained with the algorithm presented in Chapter 5 on medical data. Various applications are explored where there was a demand for the characterization of deformation between images of a same subject taken at different time points. The applications we investigate are the imaging of arm exercise, the effect of enlarging ventricles and shrinking lesions in Multiple Sclerosis (MS), and brain deformation during neurosurgery.

7.1 Volumetric Elastic Matching Results

7.1.1 Arm Data

This example studies the deformation occurring when a muscle of the arm is exercised by a two finger flexion (Fleckenstein et al., 1996). Exercise causes a muscle on the left part of the image to expand (see Figure 7.1). The aim of this exercise related muscle deformation experiment was to characterize the physical change happening during exercise, by comparing both images when they are aligned together.

In this experiment, the Young modulus of the tissue has been set to 2 kPa (and $\nu$ to 0.3), and was constant over the whole arm volume. Future enhancements should include different coefficients for the bones and the skin.
Figure 7.1: Arm exercise analysis. Slice of 3D MR dataset a) at exercise, b) at rest, c) deformation field overlayed on exercise slice, indicating deformation mainly in the area where the muscle was exercised.

The results of Figure 7.1 confirm that the muscle expansion is localized in the image area where the muscle was exercised. This is on the left part in the image, where the displacement field has the largest magnitude.

### 7.1.2 Ventricular Matching

In this experiment, the ventricles and the intra-cranial cavity of an MS patient have been segmented at two different time points (approx. 3 years apart) from 3D T2 weighted MR images.

During that period, significant enlargement of the ventricles occurred.

Figure 7.2: Illustration of the effect of enlarging ventricles of an MS patient. a) slice of difference between segmented images at both time points (gray means no difference), b) deformation field superimposed on same image at the first time point, c) close-up of b).

The matching of these time points allows us to observe the change in shape of the brain and ventricles ($E = 3kPa, \nu = 0.45$).

### 7.1.3 Lesion Growth

In Figure 7.3, we visualize the ability of our model to track the evolution of Multiple Sclerosis (MS) lesions. Our FE biomechanical matching algorithm
is able to effectively capture the change in size of MS lesions.

Figure 7.3: Matching of MS lesion growth. Close-up on slice of T2 weighted 3D MR image a) at time point 1, b) at time point 2, c) deformation field overlayed on time point 1.

### 7.1.4 Brain Shift Analysis

In this experiment, the aim was to evaluate and understand the deformation the brain undergoes during neurosurgery after the skull has been opened. This deformation will be described and studied in more detail in Chapter 9. An initial experiment was carried out to match a 3D intraoperative MR scan acquired right after the skull was opened onto a later one showing a significant shift in the direction of gravity. Figure 7.4 illustrates the deformation field our algorithm yields using 2D arrows. Note that the very different contrasts on the two intraoperative scans yielded completely unplausible deformations around the skin surface that propagate throughout the air (whose elasticity was set to be minimal though). This contrast difference can be seen in Figure 7.4c. This problem was also reported by Hata et al. (2000) who used a classical optical flow approach to match similar images. The injection of contrast agents between the scan acquisitions can also cause this kind of problems. A solution to this problem would be to work on segmentations of the object of interest i.e. brain tissue.

In another experiment, the aim was to provide the neurosurgeon with quantitative data about an intuition he had on the behavior of the brain boundary after tumor resection. He wanted to verify if brain tissue located next to the tumor boundary swells back after tumor resection. Applying our algorithm to successive intraoperative scans before and after tumor resection yielded the deformation field shown in Figure 7.5 (from Nabavi et al. (2001)), which seems to confirm his beliefs. Note that only tissue that had previously been labeled as brain has been matched in this example, the tumor was not considered in this experiment.
Figure 7.4: 3D Elastic matching of intraoperative MR images of the brain showing the propagation of the deformation at the brain surface throughout the volume. a) Axial slice of early intraoperative scan with deformation field resulting from the matching onto the later scan, b) same slice of the later scan c) same slice of difference image.

Figure 7.5: a) Axial slice of early intraoperative scan, b) same slice of later intraoperative scan, after tumor resection. a to b) Result of matching brain tissue in image a onto brain tissue in image b. The deformation field illustrates the behavior of the brain tissue in the vicinity of the tumor after resection.

7.2 Discussion

This chapter has demonstrated that an algorithm such as the one presented in Chapter 5 can effectively be used for doing elastic matching on medical images.
The visualization of the results of the experiments indicate excellent results from an image matching point of view, and the resulting deformation fields are physically plausible. Another advantage of the method is that different elasticities can be assigned to different parts of the images to be matched.

Although more results are needed to experiment and validate the method, the algorithm has several shortcomings as to our initial goal of physics-based registration and characterization of deformation:

- As already discussed when introducing the model in Chapter 5, the algorithm yields a deformation field that does not fully satisfy the constitutive equations of elastic materials as presented in Chapter 2. To solve that problem and comply with the constitutive equations of elastic materials, we should look into constrained minimization using a Lagrangian, but then one is not assured that a solution exists.

- The problem of contrast variation in the images to be matched has briefly been addressed in section 7.1.4. The algorithm assumes that both images have the same contrast and that the second image results from a deformation of the first one. When there is a contrast change from one image to the next (such as on the experiment depicted in Figure 7.4), this causes totally unplausible deformations. This problem could be solved for by matching segmented volumes to generate a deformation field that could then be used to deform the initial gray-scale image, or by using other image similarity criteria.
Chapter 8

Deformable Atlas-Based Segmentation of MR Images of the Brain

In this chapter, we use the algorithm presented in Chapter 6 for the automatic localization and identification of brain structures in MR images using a reference atlas. The deformation of the atlas onto a patient’s MR image is done in three steps: first a global registration is used to initialize key boundary surfaces of the atlas (lateral ventricles and cortical surface), next an active surface deformation is used to capture local shape differences, and finally a finite element elastic model is used to infer a volumetric mapping of the atlas onto the target image. A preliminary experiment shows excellent correspondance of the visible structures after deformation of the atlas onto a new image.

8.1 Introduction

The automatic identification and localization of structures in magnetic resonance (MR) brain images are a major part of the processing work for the neuroradiologist in numerous clinical applications, such as functional mapping and surgical planning. The accurate segmentation of brain structures is a very complex task. Manual procedures, as well as semi-automated procedures for slice by slice segmentation of 3D data are highly time consuming, while automatic procedures relying on local criteria can often hardly do more than separate the main tissue types (white matter, gray matter, CSF, lesions, etc). To aid in the identification of the imaged anatomic regions, a
considerable amount of research has been directed toward the development of 3D standardized atlases of the human brain (Talairach and Tournoux, 1988; Bohm et al., 1983; Seitz et al., 1990; Hohne et al., 1992; Thurjfell et al., 1993; Collins et al., 1994; Kikinis et al., 1996). These provide an invariant reference system and the possibility of template matching, allowing anatomical structures in new scans to be identified and analyzed.

Nevertheless, no two human brains are the same, and this presents a challenge for any attempts to create standardized atlases. Even in the absence of any pathology, neural structures will vary between individuals not only in shape and size, but also in their orientations relative to each other (Rademacher et al., 1993). The use of deformable atlases is therefore gaining increased attention in the medical imaging research community, combining the advantages of both local image analysis and global models. In particular, certain structures, such as the thalamic nuclei, and the Brodman cytoarchitectonic areas of the cortex, are not visible even in high resolution MR images. Such structures are outlined in detail in brain atlases. Therefore, mapping an atlas onto a patient’s brain morphology is of great importance. A registered atlas can then be used as a fundamental tool for the assessment of structural brain abnormalities, for mapping functional information onto the corresponding anatomy, and for computer-assisted neurosurgery.

Such model-based techniques can also dramatically decrease the time required for the localization and quantitative analysis of anatomical brain structures. It can as well improve the reproducibility and, potentially, the accuracy of the process.

For applications such as atlas matching, global transformations (same transformation is applied to every point in the dataset) cannot account for inter-subject local shape differences. Therefore, more local transformations are needed, but they have to maintain the spatial relationship between anatomical structures and allow for considerable inter-subject variability.

There are mainly two types of methods for doing deformable atlas matching: surface based and volume based methods. Surface based methods deform key surfaces of the atlas onto the target image and interpolate the surface displacement to obtain a fully volumetric mapping. Volumetric methods compute a deformation field that minimizes a similarity criterion between
the atlas and the target image under a given regularization constraint (e.g. elastic).

8.1.1 Volumetric Methods

The idea of modeling the atlas as an elastic object was originated by Broit (1981). Exploiting a suggestion of Dengler and Schmidt (1988), Bajcsy and Kovacic (1989) and Gee et al. (1993) subsequently implemented multiresolution versions of Broit’s system where the deformation of the atlas proceeds step-by-step in a coarse to fine strategy, improving the robustness of the technique. The elastically deformable volumes have become a very active area of research and have been extended to viscous fluid deformable volumes (Christensen, 1994; Christensen et al., 1997) in an attempt to overcome the small deformation assumptions inherent in the linear elastic formulations.

Recent work in this area has focussed on the use of other similarity metrics than correlation or gray-level differences. Mutual Information (MI) has been used for doing multi-modal elastic registration (see for instance Maintz et al. (1998); Hata (1998); Gaens et al. (1998)). Rueckert et al. (1999) also uses MI for free-from registration of mammograms. Thirion (1995) proposes to use ridgeness for non-rigid registration.

The main disadvantages of these methods are that, because of the need of a multiresolution approach, they have a very high computational cost (several hours are reported on a massive parallel machine for the viscous fluid model). Another issue with these methods is that they are sensitive to their initial placement, and one therefore needs to find a proper initialization prior to running the algorithm. Finally, even though such methods provide a good overall match of the atlas, mismatches of some atlas objects can still occur.

8.1.2 Surface-based Methods

Surface based methods deform key surfaces of the atlas onto the target image and interpolate the surface displacement to obtain a fully volumetric mapping of the atlas.

Davatzikos and Prince (1995) describe an algorithm for doing semi-automated deformable surface mapping of the cortex. They define specific external
forces that drive the surface toward the spine of a ribbon and integrate the
image data to fit the surface to the cortical ribbon. Davatzikos (1997) later
proposed to use an elastic warp to propagate the deformations of the cor-
tical surface inside the brain, so as to have a fully volumetric mapping.

Thompson and Toga (1996) also use a surface-based method and a subse-
quent volumetric warp to compute a volumetric deformation field. Their
surface matching algorithm proceeds in two steps: the user first needs
to identify landmark points in the target image, and parametric surfaces
(based on superquadrics and spherical harmonics) are then fit onto these
landmarks prior to running an active surface algorithm to account for local
shape differences in the target. The choice of these surfaces is based upon
developmental processes differentiating neuroanatomy between individuals
(typically, the ventricles, the cortex, and significant sulci). The final surface
deformations are then used as input to an elastic warping algorithm yielding
a fully volumetric mapping of the atlas.

McInerney and Kikinis (1998) propose a surface-based criterion, but the de-
formations of the surface directly affect the volume the surface surrounds.
The forces deforming the surface also integrate statistical image data.

The main disadvantage of surface based methods is that they typically
need a manual initialization of the deformable surfaces prior to running a
deformable surface algorithm.

8.1.3 Our Approach

Even though no studies comparing surface based and volume based de-
formable matching methods have been published so far, we believe surface-
based methods can achieve better local accuracy by adding additional sur-
face constraints to the matching rather than by performing a globally ac-
curate volumetric match.

We propose an approach combining both recognition and segmentation of
brain structures based on surface and volumetric methods. We first use a
global deformation to automatically register a surface-based model of the
brain (the atlas) to the patient’s image. The surfaces of the globally reg-
istered atlas are then used as initialization for an active surface algorithm.
To maintain a spatial relationship between the objects in the atlas, a vol-
umetric deformation field is inferred from the surface based deformations,
creating a fully volumetric warp of the atlas onto the target image.

8.2 Atlas Matching Algorithm

8.2.1 Initialization of the Deformable Surfaces - Global Atlas Registration

In (Cuisenaire et al., 1996), an automated procedure to find the best parameters for the 3D second degree global transformation of surfaces onto the Distance Transform (DT) of a target image is presented (see Section 4.2 of Chapter 4). This transformation was originally proposed and routinely used by Thurjfell et al. (1993) for registering the Computerized Brain Atlas\(^1\) (CBA) with an MR image. The registration method is based on the matching of 2 important anatomical landmarks: the cortical surface and the ventricular system.

This global transformation is equivalent to the routinely used transformation performed by the manual registration of the CBA. This part of our algorithm mainly is an automation of this procedure. Improving these results, i.e. providing a correct local shape and not only a correct localization for the brain structures, requires adding complexity to both the possible deformations and the matching criterion. We believe that by exploiting local image information, as well as prior information for visible structures in the image, it is possible to refine these results significantly.

8.2.2 Deformable Surface Matching

We match surfaces corresponding to structures that are well defined in the target image (e.g. cortical surface, lateral ventricles, etc.) using our deformable surface algorithm.

As described in Chapter 4, our active surface algorithm uses an elastic membrane model that is deformed iteratively by forces derived from the target image.

8.2.3 Volumetric Atlas Matching

As proposed by Davatzikos (1997) and Thompson and Toga (1996), having matched a few characteristic surfaces, one can thereafter infer a volumet-

\(^1\)University hospital of Karolinska, Sweden. http://cb.uu.se/~lennart/projekt\_eng.html
ric mapping of the atlas onto a new image using a volumetric model (e.g. elastic). This can be particularly useful when the target image has poor contrast, and if only few atlas structures (e.g. the cortical surface and the lateral ventricles) can be matched in the target image.

We infer a volumetric deformation field from the surface deformations by applying the surface displacements obtained after global registration and active surface deformation as a boundary condition to a volumetric FE tetrahedral mesh we have extracted from the atlas image (see Chapter 6).

After the atlas’s volumetric FE mesh has been deformed by the boundary surfaces, the obtained deformation field can be interpolated back onto the image grid to produce a volumetric mapping from the atlas space onto the target image space.

8.3 Results

We apply our algorithm on a new MR image of a healthy patient as an experiment. First, the cortical surface of the brain and the lateral ventricles are segmented out of the MR image using an algorithm inspired from the directional watershed algorithm proposed by Thiran et al. (1997) followed by mathematical morphology based operations to simplify the brain surface. Only a coarse cortical surface (without sulci) and the ventricular system are used by the matching criterion (see Figure 8.1).

![Figure 8.1: Slice of original MR image, segmentation using directional watersheds, brain contours, simplified contours.](image)

The atlas’s (we use the atlas developed by Kikinis et al. (1996)) corresponding surfaces are then matched onto the distance transformation of the segmented surfaces. The result of the matching of the ventricles and the brain surface onto the target image is shown on the first row of Fig-
8.3 Results

One can observe that the surfaces are very well localized, but that the local shape of the surfaces still differs from that of the target scan (e.g. on the top of the brain visible on the sagittal view). The active surface deformation (whose results are shown on the second row of Figure 8.2) corrects for this local shape difference quite well. Note that our algorithm still has some difficulties to capture all the shape differences, and tends to shrink the ventricles somewhat. This part of the algorithm is currently being improved, and is of course of critical importance to the accuracy of the atlas matching.

Figure 8.2: First row : cuts (a:axial - b:saggital) through deformable surfaces after 2nd degree polynomial transform overlayed on corresponding cuts through target image. Second row : same cuts (c:axial - d:saggital) but after active surface deformation.

Once these surface deformations have been computed, a volumetric deformation field can be inferred allowing images from any modality of the atlas to be warped onto the target image. Figure 8.3 illustrates this for the MR scan of the atlas. One can very well observe that the active surface deformation can account for local shape differences with the target image. However one can also notice that the active surface algorithm could not
fully recover the local shape differences of the atlas with the target image, especially at the level of the lateral ventricles.

Figure 8.3: Volumetric deformations of the atlas MR image illustrating the transformation steps of the algorithm. a) Slice of deformed volume after second degree registration of ventricles and brain surface. b) Same slice after active surface deformation. c) Same slice of target image.

The volumetric mapping can then be used to outline other objects in the target image, such as illustrated in Figure 8.4. Even though the localization of the structures is perfect, the local shape still differs somewhat. To solve for this problem, these objects could be used as well to drive the volumetric elastic deformation.

Figure 8.4: Deformation of other atlas objects overlayed on target image after volumetric transformation.

8.4 Conclusion

Our deformable matching algorithm has the advantage over most other methods that the initialization of the atlas can be done in an automated
and accurate way using a second degree polynomial transformation. An arbitrary number of surfaces can be matched to drive the volumetric warp of the atlas onto a target image, and the finite element formulation of the volumetric match allows for variable elastic deformations of the atlas’s objects. Our preliminary experiments show very good correlation of the deformed atlas with the target image. Even though the method seems to achieve good results, more experiments will be needed to validate the method and assess its accuracy.
Chapter 8. Deformable Atlas-Based Segmentation of MR Images of the Brain
Chapter 9

Deformable Registration of Intraoperative MR Images of the Brain

This chapter presents the main application we aimed at when developing our algorithms. It uses the main ideas presented in Chapter 6, but extends them to the registration of image sequences containing more than two images. In this application, the goal is to be able to register pre-operative imaging of the brain with intraoperatively acquired imaging showing the deformation of the brain during surgery. The aim here is also to be able to characterize the deformation the brain undergoes during the course of surgery, with a long term goal of doing simulation with minimal intraoperative imaging. Two experiments are presented: one on a pair of intraoperative MR images showing only gravitational shift of the brain, and a second one on a complete intraoperative sequence showing gravitational shift of the brain, two successive stages of tumor resection and a last one after dural closure.

9.1 Introduction

9.1.1 Context and Motivation

Today, image guided surgery systems are being increasingly used in the operation room (OR), and have been shown to improve surgical visualization and navigation (Jolesz, 1997), and to reduce post-surgery remaining tumor (Knauth et al., 1999). To be successful, image-guided neurosurgery relies on the registration of pre-operative imaging (MRI, CT, PET, fMRI, etc.)
with the patient’s position in the OR. This can be done using a stereotactic frame (Kelly, 1986), or with frameless registration systems (e.g. Zamorano et al. (1992); Zinreich et al. (1993)). Following this acquisition, it is possible to visualize, and navigate into the anatomy of the patient in a coordinate space consistent with the position of the patient’s brain during surgery.

However, during surgery, deformation of the brain occurs, therefore causing misalignment of the surgical planning with respect to the preoperative imaging it is based on. The major factors influencing this deformation include (Nabavi et al., 2001):

- gravity, causing a shift in that direction,
- tissue retraction and resection,
- cerebrospinal fluid (CSF) leakage and draining,
- biomechanical properties of the brain tissue,
- anaesthesiology,

Brain shift is probably due to gravity and cerebrospinal fluid (CSF) leakage or draining, and appears after the dura has been opened (Nabavi et al., 2001; Bucholz et al., 1997; Hill et al., 1998; Maurer et al., 1998). Tissue removal typically occurs in the case of tumor resection. The drugs the anaesthesiologist is administering to the patient can hydrate or dehydrate the brain, causing it to swell or to shrink (Miga et al., 1999a).

Cortical surface shifts ranging between 1.2 and 20mm have been observed (Nabavi et al., 2001; Roberts et al., 1998), while subsurface shift can range up to 6 or 7 mm (Hill et al., 1998; Roberts et al., 1998; Hata et al., 2000; Nabavi et al., 2001) at the level of the lateral ventricles and the mid-hemispheric plane.

Recently, using intraoperative MR imaging, a study (Nabavi et al., 2001) demonstrated that these deformations develop and aggregate over the course of surgery, while they remain unrecognized by navigational systems. Besides the intraoperative shift due to gravity or CSF drainage, the deformations induced by surgery (e.g. tumor resection) add up and continuously change the brain shape.
All these studies demonstrate that the accuracy of image-guided surgical navigation based upon preoperative imaging is seriously compromised over the course of surgery by intraoperative brain deformation at the cortical surface level as well as in subcortical regions of the brain. It is therefore of great importance for the image-guided neurosurgical systems to be able to correct for these deformations, analyze and characterize them, and update preoperative imaging and planning according to the shape changes of the brain during surgery. The subject has recently led to considerable interest in the medical image analysis community (Bucholz et al., 1997; Maurer et al., 1998; Paulsen et al., 1999; Hill et al., 1999; Miga et al., 1999b, 2000a; Skrinjar and Duncan, 1999; Hata et al., 1999; Hagemann et al., 1999; Ferrant et al., 2000c; Miga et al., 2000b).

9.1.2 Review of Related Work

The changes in brain shape during neurosurgery are now widely recognized as nonrigid deformations. Previous work in capturing brain deformation for neurosurgery can be categorized by those that deform preoperatively acquired imaging using image-based models and those using biomechanical models.

Most of the work that has been done in the field of intraoperative volumetric image alignment is mainly based on image related criteria (i.e. optical flow methods) (Hata et al., 1998; Edwards et al., 1998; Hata et al., 2000; Hill et al., 1999). Even though such methods have achieved reasonable success rates from an image matching point of view, they do not provide the medical doctor with physically realistic and interpretable data. Instead, doctors want to derive quantitative and physically meaningful data from the images so as to characterize deformation. Physical deformation models have already been proposed to constrain a deformation field computed from image data using elastic (Bajcsy and Kovacic, 1989; Ferrant et al., 1999b) or even viscous fluid deformation models (Christensen et al., 1997; Bro-Nielsen and Gramkow, 1996). However, these models do not account for the actual material characteristics of the objects depicted in the images, because the matching is done minimizing an energy measure that consists of a weighted sum of an image similarity term and a relaxation term representing the potential energy of a physical body (e.g. elastic). Therefore, the actual biomechanics of the phenomenon cannot be properly captured by these models, and the physical model simply acts as a regularization
constraint on the image similarity criterion.

Other authors have proposed to use deformable surface models in conjunction with a physics-based model to infer a volumetric deformation field from surface-based deformations (e.g. Davatzikos (1997); Thompson and Toga (1996)). But the used parameters in the physics-based volumetric models were determined heuristically, and one could therefore not exploit the information generated by the model to extract biomechanical properties.

Another class of registration methods using physical-based models to constrain image registration is those using statistical image information of the objects to be matched (e.g. Wang and Staib (2000); Papademetris et al. (1999a)). These methods use statistical image information, or boundary shape information to drive the deformation of the biomechanical model and appear to be less sensitive to noise, while significantly improving non-rigid registration.

The cardiac image analysis community has also been using physics-based models - mainly FE models -, but they deform them with image-derived forces. These models then provide quantitative, and physically interpretable 3D deformation estimates from image data. Papademetris et al. (1999b) derive the forces they apply to the FE model from Ultrasound (US) images using deformable contours they match from one image to the next one using a shape-tracking algorithm. Metaxas (1997) derives the forces from MRI SPAMM\(^1\) data for doing motion analysis of the left or right ventricle (Park et al., 1996; Haber et al., 1998). These approaches are very interesting and a generic parameterized FE model is usually fit onto the image sequence before doing the analysis.

It is only recently that biomechanical models have been explicitly proposed to constrain the registration of images (Kyriacou et al., 1999; Hagemann et al., 1999) in the context of deformable brain registration. Hagemann et al. (1999) use a biomechanical model to register brain images showing deformations due to neurosurgical operations. The model is deformed by enforcing correspondences between landmark contours manually or semiautomatically. The constitutive equations of the biomechanical model are

\(^1\)Spatial Modulation of Magnetization produces a regular pattern of stripes that deforms along with the imaged data. This allows for easy computation of deformation from the images using the stripe pattern.
discretized using finite elements, and the basic elements of the mesh are the pixels of the image, which causes the computations to be particularly heavy. Kyriacou et al. (1999) study the effect of tumor growth in brain images for doing atlas registration. They use a FE model and apply concentric forces to the tumor boundary to shrink it. Currently, the drawback of such methods is that they either require user intervention, or another not fully automated means to compute the forces (or correspondences) applied to the model. Another drawback is that these later methods have only been applied to 2D images thereby limiting the clinical utility and the possibility to efficiently assess the accuracy of the method.

In the context of brain shift analysis, there has been a significant amount of work directed towards simulation using models driven by physics-based forces such as gravity. Skrinjar and Duncan (1999) propose a model consisting of mass nodes interconnected by Kelvin models\(^2\) to simulate the behavior of brain tissue under gravity, with boundary conditions to model the interaction of the brain with the skull. In (Paulsen et al., 1999; Miga et al., 1999b, 2000a), the authors propose a Finite Element (FE) model based on consolidation theory where the brain is modeled as an elastic body with an interstitial fluid. They also use gravity induced forces, as well as experimentally determined boundary conditions. In a recent study (Miga et al., 2000b), an attempt towards modeling the postoperative behavior of tissue resection was done by adding a depression stress at the boundaries of the resected cavity.

Even though these models are very promising, it remains difficult to accurately estimate all the forces and boundary conditions that interact with the model, especially during the course of surgery. For instance, it is very difficult to model the shrinking of the lateral ventricles during brain shift. This phenomenon is probably due to a pressure change of the cerebrospinal fluid (CSF) inside the ventricles, but it is extremely complicated to effectively measure this pressure continuously during neurosurgery. Also, only the state of the brain before and after opening of the dura has been considered so far in most proposed models, therefore not considering the dynamic evolution of the shape of the brain during surgery. This leaves problems related to tissue retraction or removal during surgery unaddressed and unsolved.

\(^2\)A Kelvin Solid Model is a parallel connection of a linear spring and a dashpot that models a visco-elastic material subject to slow and small deformations.
9.1.3 Proposed Method

In our deformable registration method, we propose to merge the prior physical knowledge physicians have about the object that is being imaged with the information that can be extracted from the image sequence to obtain quantitative measurements. As explained in Chapter 6, we extract shape information of the objects in the image sequence using an active surface model, and characterize the changes the objects undergo using a biomechanical elastic FE model (Ferrant et al., 2000b, 2001a).

The idea is similar to that used for cardiac analysis; we track boundary surfaces in the image sequence, and we use the boundary motion as input for a FE model. The boundary motion is used as a boundary condition for the FE model to infer a volumetric deformation field. An initial FE brain model is deformed successively onto intraoperatively acquired MR images taken at crucial moments during the course of surgery (Nabavi et al., 2001). The topological changes induced by tumor resection are included into the process by removing elements from the FE mesh covering resected areas, with the purpose of modeling tissue resection, which to our knowledge has not yet been explored so far (Ferrant et al., 2001a).

Our ultimate goal is to be able to do prediction of deformation during surgery with the goal of improving intraoperative navigation and tumor resection, and of reducing the amount of intraoperative imaging that is necessary. To be able to do this, one first needs to validate a non-rigid deformation model. Intraoperative MR imaging (IMRI) provides excellent contrast and spatial resolution, which makes it an ideal testbed for developing and validating nonrigid deformation methods.

9.2 Algorithms

Figure 9.1 presents a typical sequence of intraoperative 0.5 Tesla MR images (256x256x60 matrix - 60 slices scanned from inferior to superior, voxel size 0.9375x0.9375x2.5 mm$^3$) showing brain shift due to gravity and CSF draining and leakage. The three images have been aligned so as to account for patient movement within the magnet using an algorithm based upon the maximization of Mutual Information (MI) (Wells et al., 1996; Viola and Wells, 1997). One can very well observe the shift on the third image in the direction of the gravity, as well as the shrinking of the lateral ventricles. The aim of our algorithm will be to deform the initial image onto the last
image so as to be able to transpose the surgical procedure that has been prepared preoperatively onto the target image taken during surgery when the brain has shifted.

Figure 9.1: Typical sequence of intraoperative MR scans showing brain shift due to gravity and CSF draining. The second scan was taken just after dura removal, the third scan was taken after the brain had shifted.

There are two important points for doing physics-based modeling of deformations in 3D image sequences. One first needs to have a prior biomechanical model of the object represented by the image, i.e., the constitutive equations modeling the behavior of the bodies (elastic, fluid, viscous fluid, etc.) represented in the image. On the other hand, one also needs a way of applying forces and boundary conditions to the model using the information from the image sequence.

In this work, we have chosen to model brain structures as elastic bodies, as it has been shown that soft tissue deformation can be modeled quite accurately using linear elasticity in the case of small strains (Fung, 1993; Walsh and Schettini, 1990) (see Chapter 2 for details on method to solve the constitutive equations). However, other constitutive materials such as viscous fluids, non-linear elastic bodies, etc. can of course very easily be integrated into our algorithm. Thus, we assume that the objects that are being imaged have a linear elastic behavior during deformation.

The algorithm we use is the one presented in Chapter 6. The deformations are tracked using the boundary information of the objects in the image sequence. The boundary surfaces of the initial image are deformed towards the boundaries of the next 3D image in the sequence using an active sur-
face algorithm. The deformation field of the boundary surfaces is used as a boundary condition for the volumetric biomechanical model, that will be used to infer the deformation field throughout the entire volume.

This then provides us with physically realistic and interpretable information (such as stress tensors, compression measures, etc.) of the imaged objects during the whole sequence.

The approach presented in Chapter 6 is fine for registering two images, e.g. a pre-operative (or intraoperative) onto a later intraoperative scan, but cannot describe dynamic changes during neurosurgery. Therefore, intraoperative imaging needs to be performed at crucial time points during neurosurgery (Nabavi et al., 2001) to supply more information to the model. This way, pre-operative imaging (such as DSA, CT, fMRI, etc.) can be updated more faithfully so as to reflect the shape changes of the brain during surgery. A typical complete sequence of serial intraoperative images is shown in Figure 9.2. Figure 9.2 shows the same slice of intraoperative MR images (256x256x60 matrix - 60 slices scanned from inferior to superior, voxel size 0.9375x0.9375x2.5 mm³) taken at five critical time points during surgery (see Nabavi et al. (2001) for method) after rigid alignment using mutual information (Viola and Wells, 1997; Wells et al., 1996). The first scan was taken at start of surgery, the second one after the dura had been opened and some CSF drained, showing significant brain shift in the direction of gravity (downwards), the third and fourth scans show 2 stages of tissue retraction and tumor resection also causing the brain to slightly shift in the direction of gravity, and the fifth scan shows the brain swelling up again. Note that a contrast agent³ had been injected only after the first scan was taken to highlight the tumor.

Figure 9.3 depicts how one can do serial registration of intraoperative images during surgery using the same ideas as those presented in Chapter 6, but with multiple images.

In Figure 9.3, all the blocks that are on the left of the dashed vertical line show tasks that can be executed pre-operatively, and hence more time is available to perform them. The tasks to be executed preoperatively include the extraction from pre-operative imaging of the objects the neurosurgeon wishes to track and characterize over the course of surgery (typically the

³Gadolinium (III) - diethytriaminepentaacetic acid (DTPA).
brain, the tumor, and the ventricles), and the generation of a FE tetrahedral model of these objects that will be deformed during surgery to match their shape changes. Different biomechanical properties could be assigned to the different objects and the boundary surfaces of these key objects can be extracted from the model. On the right side of the dashed vertical line of Figure 9.3, every column depicts the different operations to be carried out during neurosurgery each time a new intraoperative MR scan is acquired. The different boxes are executed from top to bottom: first an intraoperative segmentation of the scan is done to extract the key objects, then the boundary surfaces of the model are deformed to match the shape changes the segmentation reflects, these deformations are eventually used to infer a volumetric deformation field with the biomechanical model, which can be used to update the preoperative scan(s). This sequence of operations depicted by the vertical sequence of boxes is repeated each time a new intra-operative scan is acquired.

Tissue retraction and resection introduce topology changes or major shape
modifications for the brain, since tissue is being cut and removed. This aspect is being taken into account by removing elements of the finite element (FE) model covering retracted and resected areas so as to reflect the changes. The areas of resection appear as black holes within the brain tissue image and are thus easy to segment out of the intraoperative images. Those elements of the FE model remaining on the resected areas between the tumor boundary surface and the brain surface are removed to account for the topology change introduced by cutting the brain tissue. Special care has been taken to ensure that the model remains consistent with respect to its earlier configuration, so that deformation can still be characterized from the model at previous time points.

Our algorithm now consists of seven main components:

- **Preoperative segmentation**: key objects whose shape will change over the course of surgery are precisely segmented out of preopera-
9.2 Algorithms

- **Tetrahedral mesh generation**: an initial FE model of the brain is generated using the preoperative segmentation as explained in Chapter 3 (Ferrant et al., 1999b, 2000a),

- **Intraoperative segmentation**: the key objects are segmented out intraoperatively each time new imaging is acquired using the algorithm described by Warfield et al. (2000c),

- **Deformable surface matching**: the boundary surfaces of the key objects in the model are deformed towards the corresponding boundaries in the next image of the sequence as explained in Chapter 4 (Ferrant et al., 1999a, 2000a).

- **Biomechanical simulation of volumetric deformation**: the obtained displacements of the surfaces are used as boundary conditions to drive our biomechanical model (Ferrant et al., 2000c,b) (see Chapters 2 and 6 for method).

- **Interpolate deformation field and deform image**: the deformation field of the FE model is interpolated back onto the image grid to deform pre-operative image data so as to update the preoperative planning on the current shape of the brain (Ferrant et al., 2000c,b).

- **Tumor resection modeling**: at the stage at which tissue retraction starts, based on the intraoperative segmentation of retracted and resected areas, the topology of the FE model is updated so as to reflect the topology changes this induces.

Steps 1 and 2 are executed pre-operatively and can be more time-consuming, while steps 3,4,5,6 are executed during the course of surgery, and repeated each time a new scan is acquired. Step 7 is executed each time resection or retraction significantly modifies the shape or topology of one of the key objects in the model.

The following sections briefly describe the elements of our algorithm that have not yet been reviewed in the previous chapters.

### 9.2.1 Preoperative and Intraoperative Segmentation

In order to be able to successfully track and characterize the deformation of the key objects the neurosurgeon is interested in (e.g. brain, tumor
and lateral ventricles) during surgery, we need to segment them out of the intraoperative images. This task is extremely difficult because the intraoperatively acquired data has limited resolution and contrast, and it is therefore often difficult to distinguish between tissue types.

Since preoperative data is acquired before surgery, the time available for segmentation is longer. This means we can use segmentation approaches that are robust and accurate but are time consuming and hence impractical to use in the operating room (Warfield et al., 2000b; Kaus et al., 2000; Warfield et al., 1995). The segmented preoperative data can then be used as a patient-specific atlas which enables robust and real-time segmentation (Warfield et al., 2000c, 1998) of the intraoperative images, that fits within the time constraints of neurosurgery.

9.2.2 Modeling Tissue Resection

During neurosurgery, the surgeon often needs to retract and resect tissue, for instance for removing tumor. Practically, the surgeon retracts the brain, bringing neighboring tissue under constraint. Tumor tissue is then resected until no more remains. This means that the shape and topology of the brain change after tumor resection starts. This change needs to be included into our model so as to be able to characterize further deformations faithfully.

The way we cope with this topology change is by updating our model so it reflects the changes at the time point where they appear. We do this by clipping the existing mesh model (deformed onto the shape of the brain at the time point where resection appears) at the boundary of the resected areas. The clipping is done the same way as at the meshing stage: tetrahedra lying inside resection areas are removed from the mesh, and those lying across the boundary between brain and non-brain are cut so as to have a precise boundary representation of the brain with the resected areas. This operation thus reflects what the surgeon does: the surgeon displaces or retracts overlying parts of the brain creating a pathway to the tumor, which he then progressively resects. After that stage, in our model, the tumor boundary and the brain boundary merge and form one new single boundary surface.

To ensure that we are able to link with the model at the previous time point, our topology modification algorithm keeps track of the correspondence between elements and nodes of the new resected FE model with
the FE model before removal of the elements. This is essential for further characterization of the deformation.

9.3 Results

In this section, we report on results we obtained on two intraoperative sequences of MR images. The first one (see Figure 9.1) illustrates brain shift related only to gravity and CSF draining and leakage. The second one (see Figure 9.2) illustrates a complete intraoperative sequence including tumor resection as well as shift due to gravity and CSF draining, and swelling after tumor resection.

9.3.1 Material Properties

The material equations we use to model brain deformation are those of linear elasticity, as it has been shown that soft tissue deformation can be modeled quite accurately using linear elasticity in the case of small strains (Fung, 1993; Walsh and Schettini, 1990). An isotropic linear elastic material is characterized by two parameters: Young’s elasticity modulus $E$ and Poisson’s ratio $\nu$ (Zienkiewicz and Taylor, 1987). They determine the elastic behavior of the object. The choice of these values is of course critical to the reliability of a physics based deformation model. Their determination has not been addressed very consistently in the literature as the coefficients used often differ significantly from study to study and do not always include their physical units. Recently, Hagemann et al. (1999) published a comparative study of brain elasticity coefficients proposed by different authors, and came to the conclusion that for their application, the only comparable and meaningful values presented by other authors are the ratios of the coefficients for brain and skull. Since we are only interested in modeling the brain, and not the skull, we have chosen to use parameters similar to those Miga et al. (2000a) obtained with in-vivo experiments instead ($E = 3kPa, \nu = 0.45$).

9.3.2 Registration of Two Intraoperative MR Images of the Brain

FE Model Generation

The key object during neurosurgery is of course the brain. It is its shape changes that the neurosurgeon wants to be able to track. As CSF drainage
and/or leakage occurs during surgery, the boundary with the ventricles also needs to be included into the model. As reported earlier (Ferrant et al., 2000c,b; Hata et al., 2000; Nabavi et al., 2001), significant shifts and CSF volume losses have been reported at the level of the ventricles, hence it is important to segment these out of the intraoperative image sequence as well. The segmented volume was further simplified using mathematical morphology to obtain a smooth surface. Figure 9.4 shows cuts through a sample tetrahedral mesh of the brain overlayed on the corresponding initial image. Note that the mesh has been adaptively refined only in the neighborhood of the lateral ventricles, so as to ensure sufficient resolution of the surfaces for the active surface algorithm.

![Figure 9.4: Axial, sagittal, and coronal cuts through tetrahedral mesh of the brain with refinement in the vicinity of the lateral ventricles overlayed on corresponding cuts through preoperative image.](image)

**Active Surface Matching**

The active surfaces are extracted from the intraoperative scan at the start of surgery, before opening the dura mater (see Figure 9.5a), and deformed towards the brain in a later intraoperative image (see Figure 9.5b). One can very clearly observe that the deformation of the cortical surface is happening in the direction of gravity and is mainly located where the dura was removed. Also, one can observe a shift, as well as a contraction of the lateral ventricles. Figure 9.7 shows the 3D surface deformation field the brain and the ventricles have undergone. One can very well observe that the shift is mainly affecting the left part of the ventricles, while the displacement of the lower parts is mostly due to volume loss.
9.3 Results

Figure 9.5: Illustration of the capture of the boundary surface deformation using our active surface algorithm. Axial cut through initial (a,b) and deformed (c) active surfaces overlayed on corresponding slice of a)initial; b,c)target intraoperative MR image.

Volumetric FE Deformation

The deformation field obtained with the active surface algorithm is then used as input for our biomechanical FE model. The algorithm yields a deformation vector for every node of the mesh. These displacements can then be interpolated back onto the image grid using the shape functions within every element of the FE mesh (see Eq. 2.7). Figure 9.6 shows a slice of the deformed image as well as the image of the difference with the target. One can observe that the algorithm captured the surface shift and the ventricular thinning very accurately. The square root of the gray-level mean square difference between the target scan and the deformed original scan on the image regions covered by the mesh went down from 15 to 3. However, one can also notice that the left ventricle (lower one on the Figure) was not able to fully capture the thinning. This is due to the approximate model of the lateral ventricles we used in this experiment.

Figure 9.8 shows orthogonal cuts through the target intraoperative scan with transparently overlayed color-coding of the intensity of the deformation field. The arrows show the actual displacement of the nodes of the mesh. The extremely dense vector field in the neighborhood of the lateral ventricles is due to the adaptive refinement of the mesh at these locations.

Performance Analysis

Figure 9.9a shows the obtained deformation field overlayed on a slice of the initial scan, and Figure 9.9b shows the same slice of the initial scan deformed with the obtained deformation field. Several landmarks have also
been placed on the initial scan (green crosses) and deformed onto the target scan (red crosses), and these last landmarks have also been overlayed on the target scan for comparison with the actual deformed anatomy.

Similar landmarks as those shown in Figure 9.9 have been placed on 4 different slices where the shift was most visible, and the distance between deformed landmarks and target landmarks (not represented here for better visibility) have been measured. The surface based landmarks on the deformed scan were within 1mm of the landmarks on the target intraoperative scan. The errors between the landmarks placed in between the mid-sagittal plane and the cortical surface were within 2-3mm from the actual landmarks. The largest errors were observed at the level of the mid-sagittal plane and ventricles, which can be explained by the fact that the surface matching of the ventricles was not perfect. Nevertheless, the algorithm re-
9.3 Results

Figure 9.7: Results of boundary surface deformation. 3D surface renderings of active surfaces with color-coded intensity of the deformation field. a) brain surface, b) lateral ventricles.

Figure 9.8: Illustration of the propagation of the boundary surface deformation through the elastic brain volume. 3D Volumetric Deformation field (downsampled 12x, scaled 2x) with orthogonal cuts through target intra-operative MR image and transparently overlayed color coded intensity of the deformation field. a) Axial view, gravity is downwards. b) Coronal view, gravity goes from left to right.
Figure 9.9: Assessement of the accuracy of the registration algorithm using landmarks. a)Volumetric deformation field and initial landmarks (green) overlayed on initial intraoperative image slice. b) Same slice of deformed initial image with deformed initial landmarks (red). c) Same slice of target image with deformed landmarks.

duced the distance between landmarks in the initial and the target scans from up to almost 1cm to less than 1mm for the surface-based landmarks, and from up to 6mm to 3mm or less for the sub-surface landmarks.

9.3.3 Serial Registration of Intraoperative MR Images of the Brain

Preoperative and Intraoperative Segmentation

As in the previous experiment, we segment out brain tissue as well as the lateral ventricles. Since tumor tissue will be resected, the surgeon is particularly interested in brain tissue surrounding the tumor. Therefore, the tumor will also be segmented out of the image sequence. We thus will consider 3 key objects in this experiment: brain tissue, tumor, and ventricular CSF.

The segmentation of the intraoperative scans (Warfield et al., 2000c) is done to extract the key objects we are interested in. Figure 9.10 shows the result for the slices corresponding to those in Figure 9.2. Since no contrast agent had been injected before the first scan was taken, the tumor is not clearly visible in the first intraoperative scan, so the tumor segmentation of a corresponding preoperative scan was used.
9.3 Results

Figure 9.10: Illustration of intraoperative segmentations. Axial slice as in Figure 9.2 showing intraoperative segmentations of the scans taken during surgery.

**FE Tetrahedral Model Generation**

Next, a multi-resolution FE tetrahedral mesh model is generated from the pre-operative high-resolution segmented data. In this study, the initial coarse mesh has been refined at the location of the ventricles and of the tumor so as to ensure sufficient precision for matching these structures throughout the sequence. Figure 9.11 shows orthogonal cuts along the coordinate axes through the generated tetrahedral mesh overlayed on the corresponding cuts through the gray-level MR scan.

Figure 9.11: Orthogonal cuts through the tetrahedral mesh model overlayed on corresponding cuts through the gray-scale MR image.

**Deformable Surface Tracking**

The boundaries of the key structures can now be tracked all over the sequence. The initial surfaces are extracted from the initial FE model and successively deformed towards the corresponding boundaries of the objects in the intraoperative segmented image sequence using our active surface algorithm. Figure 9.12 shows cuts through the 3D deformable boundary surfaces at the different time points during surgery overlayed on the cor-
responding cuts through the intraoperative MR images. One can clearly observe the cortical surface shift aggregating in the direction of gravity until the 4th scan, and slightly reacting in the opposite direction from the 4th to the 5th scan.

Figure 9.12: Axial slice and corresponding cuts through deformable boundary surface. From left to right, top to bottom: (a) 1st scan with initial boundary surfaces, (b) 2nd scan with same surfaces, (c) 2nd scan boundary surfaces deformed onto scan 2, (d) 3rd scan with same surfaces, (e) 3rd scan with deformed surfaces, (f) 4th scan with same surfaces, (g) 4th scan with deformed surfaces, (h) 5th scan with same surfaces, (i) 5th scan with deformed surfaces.

Figure 9.13 and Figure 9.14 show the deformation of the brain and ventricular boundary surfaces from one time point to the next over the entire course of surgery in 3D. The surface renderings are colorcoded according to the intensity of the actual deformation. The 3D arrows represent the deformation field the surface has undergone. From time point 1 to time
9.3 Results

Figure 9.13: 3D surface renderings of the cortical surfaces with color coding of the deformation and 3D arrows showing the actual deformation from one scan to the next. a) represents the brain deformed onto scan 2 with deformation field from the 1st scan to the 2nd, b) brain 3 with deformation field from scan 2 to 3, c) brain 4 with deformation field from scan 3 to 4, d) brain 5 with deformation field from scan 4 to 5, e) brain 4 with deformation field from scan 4 to 5 (arrows are scaled 2 times here).
Chapter 9. Deformable Registration of Intraoperative MR Images of the Brain

Figure 9.14: 3D surface renderings of the lateral ventricular surfaces with color coding of the deformation and 3D arrows showing the actual deformation from one scan to the next.

Figure 9.15: 2D axial (a,b) and coronal (c) cuts through topologically adapted boundary surface of the brain (red) and ventricles (yellow) overlayed on corresponding cut through 3rd MR scan.
9.3 Results

Point 2 (Figures 9.13a, 9.14a) after dural opening, a significant shift of the upper part of the brain (ranging between 4 and 7.5 mm) can be observed in the direction of gravity. This shift propagates down to the lateral ventricle located in the upper hemisphere of the brain (shift ranging between 1.5 and 3 mm). This downward shift aggregates and amplifies both in amplitude and in extension as tumor resection starts and progresses. From time point 2 to time point 3, tumor resection starts along with an extension of the previous shift both in amplitude (2 mm in average for the brain, 2 mm for the ventricles) and extension (see Figure 9.13b). As tumor resection progresses towards time point 4, the phenomenon slightly expands further (see Figures 9.13c and 9.14c). From time point 4 to time point 5, the brain reacts and swells up in the opposite direction to the previous shifts. Figure 9.13d shows the brain surface at time point 5 with the deformation arrows showing the deformation from the brain surface at time point 4 to time point 5. The area swelling up from time point 4 to time point 5 is shown in Figure 9.13e: the brain surface shown here is the one at time point 4, with the deformation arrows showing the swelling towards time point 5. At the level of the lateral ventricles, no consistent behavior could be observed (see Fig. 9.14d), the deformations observed are probably due to inaccuracies in the intraoperative segmentation, and a poor contrast on the fifth intraoperative scan.

The boundary surface of the tumor is not deformed independently until tumor resection starts. Before that, the tumor is included into the brain model and treated identically to normal brain tissue.

Modeling Topological Changes for Tissue Resection

From the scan at time point 2 till time point 3 a topology change occurs due to tissue retraction and resection. This can be seen on the scans as black areas within the brain (see for instance Figure 9.15a). To model the tissue resection, the FE mesh deformed onto the scan showing resection (3rd scan) is clipped and elements covering the resection areas are removed so as to reflect the surgically induced topological change. Practically, this means that the brain boundary now expands into the tumor area through a small pathway. Figure 9.15 illustrates this on 2D cuts, while Figure 9.16 shows 3D surface renderings of the topologically changed boundary surface of the brain. The hole created by the tissue retraction can very well be observed.
Figure 9.16: a) 3D surface rendering of the brain surface after topological change modeling tissue resection. b) Surface rendering of the same surface cut along a sagittal plane showing the inside of the brain boundary surface at the level of the resection and tumor areas.

The modified surface is then deformed onto the next scans as shown in Figure 9.12. Figure 9.17 shows the results of the matching at the next time points on the corresponding axial and coronal cuts through the intraoperative images.

**Inferring a Biomechanical Volumetric Deformation Field from Surface Deformations**

Once the boundary surface deformations have been computed, we use them as a boundary condition to our biomechanical FE model. Solving for the deformation field with Equation 2.14 provides us with a volumetric deformation field at every node of our FE model. Figure 9.18 shows the color-coded 3D deformation field from one image of the sequence to the next. It can be clearly observed that the deformations at the boundary surfaces computed in the previous section propagate into the brain volume. From time point 1 to time point 2 the sinking propagates from the brain surface down to the mid-sagittal plane, also affecting the left lateral ventricle. The same applies until time point 4. In the areas surrounding tumored tissue and resection areas, the deformation arrows show the reaction of the brain tissue after resection. The swelling of the brain is difficult to see on the 4th image of Figure 9.18 because it is of the order of magnitude of the deformations caused by inaccuracies in the segmentations of the other areas.
of the cortical surface. The deformation field has been clipped to 2.5 \textit{mm} on that image so as to highlight the swelling of the brain. A better way to visualize the deformation field is to look at the deformation field on 2D slices. This will be described in the next section.

**Updating Preoperative Imaging**

The volumetric deformation field of the FE model can then be interpolated back onto the image grid using the shape functions on every element of the mesh. This way, it can be used to deform pre-operative imaging, so the surgeon can have an updated view of pre-operative planning he had done before hand.

Figure 9.19 shows the 2D deformation field from one image to the next overlayed on the image the deformation originates from. This provides the doctor with a comprehensive way to visualize the deformation the brain is undergoing from one time point to the next. Enlargements of regions where the deformation field is less visible are also shown. As discussed previously, the deformation field propagates below the brain surface up to the left lateral ventricle. Until time point 4, the brain shifts in the direc-
Figure 9.18: Orthogonal cuts along the coordinate axes of the color-coded FE model at the different imaged time points (2 to 5) during surgery transparently overlayed on corresponding cuts through intraoperative MR images. The arrows represent the deformation field, subsampled 5 times, and scaled 2 times.
9.3 Results

tion of gravity (e.g. downwards in Figure 9.19), increasing progressively. From time point 3 to time point 4, a lateral shift can also be observed. This lateral shift may be caused by tumor resection. From time point 4 to time point 5, the brain slightly swells in the opposite direction to gravity.

Figure 9.19: Top row: 2D deformation field overlayed on axial slice at different time points (1 to 4) during surgery. Bottom row: enlargements of the same deformation field on the same slice.

In Figure 9.20, enlargements of deformation fields in regions neighboring the tumor and resected areas are shown. One can observe that the brain swells towards the interior of the resection cavity. On the bottom row of Figure 9.20 showing the deformation field from scan 4 to scan 5, the swelling of the cortical surface can be seen along with the shrinking of the boundary of the brain with the tumor and resected areas.

Figure 9.21 shows the same axial slice as in Figure 9.2 of the gray-level images updated with the obtained deformation field throughout the sequence (from time point 2 to time point 5). After the update of the model at time point 3, no image data from the previous time point could be updated because the model was not covering the tumor and resection areas anymore. This explains why the tumor and resection areas show up bright on the two last images, on both the top and the bottom rows.
Figure 9.20: Top row : enlargements around tumor and resection area of 2D deformation field from scan three to scan 4 on three slices (38,43, and 47) of scan 3. Bottom row : same slices, but on scan 4 with deformation field from scan 4 to scan 5.

Figure 9.21: Top row : axial slice of deformed images. Bottom row : same slice of difference image of deformed with target images.
Characterization of Deformation

To characterize and better understand the deformation the brain undergoes during surgery, the visualization of the stress and strain tensors can be very useful. Figure 9.22 shows the stress tensors in 3D after deformation of the model at the different time points (2 to 5).

Figure 9.22: 3D visualization of stress tensor data. The colorcoding corresponds to the largest eigenvalue of the stress tensor at every node of the mesh. The stress tensors are subsampled by a factor 10 and represented using ellipsoids whose axes are scaled using the eigenvalues of the stress tensor.

The colorcoding is done according to the largest eigenvalue of the stress
tensor at every node of the FE mesh. Positive values correspond to a state of compression, while negative correspond to tension. The ellipsoids show the components of the stress tensor: their major axis corresponds to the eigenvector of the largest eigenvalue, the minor axis to the eigenvector of the smallest eigenvalue, and the medium axis to the remaining eigenvector. These visualizations help us better understand the load conditions of the brain during surgery. Figure 9.22 confirms that from time point 1 to time point 4 the brain is being compressed progressively on an expanding area (red and yellow regions in Figure 9.22), and slightly decompresses from time point 4 to time point 5 on the entire compression area (light blue in Figure 9.22).

Figure 9.23: Top row: axial slice of intraoperative MR images with manually placed landmarks. Bottom row: same axial slice of deformed images with deformed landmarks.

Performance Analysis

In order to assess the accuracy of our algorithm and the validity of our deformation model, we have manually placed sets of landmarks on the different intraoperative scans and tracked them using our algorithm. Thirty-two landmarks have been placed on 4 slices (axial slices 32, 35, 38, and 41).
where the brain deformation was most significant, and divided into those located on the brain surface (upper), subsurface landmarks (middle) and mid-sagittal landmarks (central). The landmarks have been placed in the upper hemisphere of the brain on every intraoperative scan.

The first row of Figure 9.23 shows the manually placed original landmarks on a slice of the initial intraoperative scan. The second row of Figure 9.23 shows manually placed landmarks on the corresponding axial slice of the four later intraoperative scans, while the bottom row shows the landmarks displaced with the deformation field generated by our algorithm. One can observe they match the manually placed landmarks in the target images quite well. The distance of the deformed landmarks to the manually placed landmarks has been measured, the results are reported in Table 9.1.

Table 9.1: Average distance in mm between deformed landmarks and manually placed landmarks.

<table>
<thead>
<tr>
<th>Time point</th>
<th>upper</th>
<th>middle</th>
<th>central</th>
<th>total</th>
</tr>
</thead>
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<td>1.80</td>
<td>3.12</td>
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</tr>
<tr>
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<td>1.87</td>
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</tr>
<tr>
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<td>1.96</td>
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</tr>
<tr>
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<td>0.66</td>
<td>2.60</td>
<td>3.73</td>
<td>2.79</td>
</tr>
<tr>
<td>Mean</td>
<td>1.22</td>
<td>2.17</td>
<td>2.88</td>
<td>2.53</td>
</tr>
</tbody>
</table>

The average distance of all the displaced landmarks to the manually placed landmarks was 2.5 mm, which can be considered as quite accurate given the imprecision of the manual landmark picking. One can notice that the best accuracy was obtained for the landmarks located on the brain surface, and the worst results were obtained for the central landmarks, in the neighborhood of the ventricles. This inaccuracy is due to the limited accuracy of our active surface algorithm in matching the lateral ventricular surfaces at some locations. Nevertheless, our algorithm reduced the distance between landmarks from up to almost 8 mm before deformation down to 1 mm for surface-based landmarks, and significant improvements were also observed for subsurface landmarks.

9.3.4 Timeline Analysis of Deformable Registration

All the components of our non-rigid deformation and tracking algorithm were parallelized so as to be able to provide results to the surgeon within
the time constraints of neurosurgery. Figure 9.24 presents a typical timeline of the image processing, analysis and simulation tasks executed during neurosurgery. It must be noted that the image analysis and simulation tasks are altogether taking up a time which is of the same order of magnitude as that of the intraoperative image acquisition. This timeline analysis demonstrates that with the parallel implementation of our algorithms, the intraoperative image analysis methods are sufficiently fast to provide feedback to the surgeon at a rate which is practical to use during neurosurgery.

![Timeline of image processing for image guided neurosurgery](image)

Figure 9.24: A timeline of typical intraoperative image acquisition and analysis tasks.

### 9.3.5 Discussion

Our algorithm is capable of characterizing and tracking intraoperative brain deformation quite accurately. However, certain aspects of the algorithm need to be discussed.

The **accuracy** we obtain for the **registration** depends on that of the different components of our algorithm:

- **The intraoperative segmentation**: given the resolution of the data, a segmentation error of 1 voxel in the slice acquisition direction can result in a bogus deformation of 2.5 mm, which explains, for instance, some of the implausible deformation obtained at the bottom of the brain. However, because the voxel-size in the direction of gravity is 0.9375 mm, we are able to track deformations of the order
of 1 – 2 mm, such as those showing the brain swelling from scan 4 to scan 5. It must be noted that in the case analysed, the intraoperative segmentation of the left (upper on the axial slices) cortical hemisphere was particularly accurate. The presence of large sulci in the other hemisphere caused some inaccuracies and inconsistencies from one scan to the next in the segmentation, therefore also affecting the surface matches causing errors to propagate throughout the hemisphere. Nevertheless, these errors do not affect the accuracy of the matching in the left hemisphere where the surgical procedure was carried out, and hence are of less importance for surgical guidance.

- **The active surface deformation**: the active surface algorithm performs very well on smooth surfaces such as the simplified cortical surface we use, but on more complicated surfaces such as the lateral ventricles, the algorithm has difficulties capturing sharp folds such as the horns (see Figure 9.12). We are currently improving our algorithm.

There are also a number of problems related to the **biomechanical FE model** itself:

- **The biomechanical model**: the model we currently use is limited to linear elasticity. For larger displacements, we intend to investigate non-linear elasticity as described in (Picinbono and Ayache, 2000). Also, we currently consider the brain as an homogeneous body, while tumor and other structures can have a very different biomechanical behavior that our model currently does not incorporate. For intraoperative use, modelling extra structures would require a real-time intraoperative segmentation able to robustly identify them.

- **Tetrahedral flipping**: due to badly shaped elements, the FE deformation field resulting from the prescribed surface based deformations can cause deformation of the elements that cannot be captured by the straight edges of the elements. Tetrahedra having a bad aspect ratio are more likely to exhibit such a behavior. The result of such deformations is that the tetrahedra end up flipped, and when updating preoperative imaging, this causes black boxes to appear on the reconstructed image, because it was not possible to find interpolated image grid points from flipped tetrahedra.

- **Tissue resection and retraction**: tissue resection represents tissue removal, and therefore our algorithm models this correctly by
removing elements from the model. However, it must be noted that the pathway leading to the tumor for instance is not resected but just retracted. For maximum accuracy, this would require tracking the surgical instruments intraoperatively, but this data is currently not collected at our institution. This should be modeled as well to faithfully capture the deformation of the brain.

### 9.4 Conclusions and Perspectives

In the two experiments presented in this chapter, we have tracked and characterized brain deformation using a fully 3D finite element biomechanical model during the course of neurosurgery. Deformations in the two presented cases were analysed extensively.

To our knowledge, this is the first time deformations of a biomechanical model are assessed with serial volumetric intraoperative acquisitions, analysing the actual behavior of brain tissue during neurosurgery (Nabavi et al., 2001).

The results of our algorithm show that the brain surface progressively sinks during surgery, and swells back after tumor resection is complete (observed on the second case analysed). Moreover, cortical shift propagates to subcortical areas, down to the ventricles (on both cases). Much more complicated deformations were observed on brain tissue around the tumor and resected areas. The brain has been observed to swell at the boundaries with the tumor and resection areas, shrinking them (on the second case).

Our study demonstrates that one cannot interpolate deformation from preoperative and single intraoperative (or postoperative) imaging to understand what happens during surgery. It is necessary to perform imaging at crucial timepoints (Nabavi et al., 2001) during surgery to understand how the brain deforms.

One of the advantages of our method is that thanks to the fully volumetric deformation field the algorithm produces, other imaging modalities acquired preoperatively (such as PET, CT, fMRI, DSA, etc.) can be updated and viewed during surgery as well.

The areas we are planning to further investigate are to include tissue het-
erogeneity, which should be straightforward within the finite element formulation. With more complete intraoperatively computed segmentations, it will be easy to assign different material properties to the elements of the mesh covering the different parts of the brain. Other possible areas of investigation are the modeling of the falx, which has been observed to play a role similar to that of a barrier against which brain tissue slides or gets compressed. However, as discussed by Miga et al. (2000a), there is only limited data in the literature reporting on differences in mechanical properties of brain tissue. We believe serial IMRI acquisition and biomechanical modelling may offer a mechanism to estimate these biomechanical parameters in the future.
Chapter 9. Deformable Registration of Intraoperative MR Images of the Brain
Chapter 10

Conclusions and Perspectives

In this chapter, we summarize the major achievements and contributions of this thesis, and suggest directions for future research in the area, both from an algorithmic and an application point of view.

10.1 Major Achievements and Contributions

This work was driven by the belief that deformable registration and characterization of deformation in 3D medical image sequences could benefit from the use of physics-based modeling (PBM).

Specifically, we developed a generic framework for doing PBM of medical structures for tracking, visualizing, and registering 3D image sequences. We used physics-based material equations (elasticity), and the Finite Element Method for discretizing the modeling equations. An original method combining surface-based deformable surface tracking and volumetric deformation models was used for registration, as well as for tracking and characterizing deformation in image sequences.

The PBM algorithms presented have been successfully applied to original medical imaging data, and have led to interesting and concluding results. We demonstrated that the approach we presented would be suitable for use within the time constraints of neurosurgery in the case of brain shift analysis for instance. This was achieved thanks to special efforts in designing optimized software for our algorithms.
The major algorithmic contributions of this work are:

- An original algorithm combining deformable surface models and volumetric FE elastic models for doing registration and characterization of deformation from image sequences (Chapter 6).

- A fast, effective and original tetrahedral FE meshing algorithm, capable of generating tetrahedral meshes with multiple embedded domains and easily controllable resolution (Chapter 3).

- A fast and parallelized implementation of the Finite Element deformation of elastic volumes given input forces and boundary conditions (Chapter 2), so that brain deformations can be computed within the time constraints of neurosurgery.

Major applicative contribution:

- Significant contributions to the analysis of brain deformation during neurosurgery using the previously mentioned algorithm. More specifically, to our knowledge, this is the first time tumor resection and brain swelling afterwards are analysed using a biomechanical model in conjunction with intraoperative imaging.

10.2 Perspectives and Future Work

The work presented in this thesis concerned the development of a PBM framework for doing registration and characterization of deformation in image sequences showing brain deformation.

At the algorithmic level, there are several improvements and extensions I suggest:

- **Material modeling**: in the framework we presented, we limited the material properties to linear elasticity. We suggest investigating different material equations that better model the behavior of brain tissue (such as proposed by Miller (1999) for instance), especially if one considers dynamic simulation with minimal input to the initial model.

- **Better surface matching**: with the active surface algorithm we presented, there is no guarantee that the obtained deformation exactly depicts the physical deformation the surface of the object has
undergone. Therefore, looking into better, and more faithful surface matching algorithms (such as proposed by Jaume et al. (2001)) would benefit to the algorithm.

- **Mesh improvement**: to avoid tetrahedral flipping (see section 9.3.5 of Chapter 9), one should look into improvement of element shape for tetrahedra having bad aspect ratios. This would speed up the matrix system resolutions and avoid problems related to tetrahedral flipping. Also, an evaluation of the error due to the resolution of the finite element discretization used should be done so one can decide whether the FE mesh should be refined in some areas or not to obtain more accurate results.

- **Better modeling**: for doing simulation with less intraoperative imaging, instead of just relying on intraoperative imaging, one should better model the behavior of the brain during surgery. For instance, one could model the interaction of the brain with the skull by adding proper boundary conditions (e.g. as proposed by Skrinjar et al. (1998)), modeling CSF pressure to be able to faithfully capture ventricular deformation, model the falx (Miga et al., 1999a, e.g.) as well as tissue heterogeneity (as suggested in section 9.3.5 of Chapter 9), while still using intraoperative imaging (or another imaging modality, e.g. US) to partly drive the model and validate it.

At the application level, I believe a number of extensions and applications should be investigated:

- **Validation**: although the error analysis indicates excellent results, obviously, the algorithm needs to be tested on a larger number of cases to validate the method.

- **Tumor modeling**: there are very different types of tumor, whose behavior can significantly vary from case to case. Some do not affect neighboring brain tissue, but others bring it under constraint, and cause the brain to swell right after the dura has been removed. Modeling the constraints different kinds of tumors induce on neighboring tissue depending on their size, etc. is a potentially very interesting area of research that will benefit to the simulation of brain shift during neurosurgery.

- **Modeling tissue retraction**: for reliable modeling and simulation, this issue should be investigated properly by collecting appropriate
data during surgery. However, so far, to our knowledge, only very preliminary data (Platenik et al., 2001) has been reported on this.

- **Extension to different applications**: the algorithm could be applied to other data than just brain. Prostate matching has been tried out, I would suggest trying out the algorithm on real-time MRI of the heart for instance. Any other application involving serial imaging of soft-tissue deformation could be considered.

### 10.3 Related Publications

The work carried out during this thesis has led to a number of different algorithms and applications that have been published at international conferences. Some results have not yet been published or even submitted. Extended versions of the conference papers have been submitted to international journals.

The active surface algorithm presented in Chapter 4 with the part of the results presented in Chapter 8 concerning deformable surfaces has been published (Ferrant et al., 1999a) and presented as a poster at the *SPIE Medical Imaging 99* image processing conference in February 1999 in San Diego, CA, USA.

The algorithm on elastic image matching presented in Chapter 5 and part of the applications shown in Chapter 7 have been published (Ferrant et al., 1999b) and presented as a poster at the *MICCAI 99: Medical Image Computing and Computer Assisted Intervention* conference in October 1999 in Cambridge, England. The basic ideas describing our tetrahedral meshing algorithm were also presented at this conference.

The algorithm combining active surfaces and volumetric FE models, along with a complete description of the FE tetrahedral meshing algorithm, were published (Ferrant et al., 2000a) and presented at the *DGCI 2000: Discrete Geometry for Computer Imagery* conference in December 2000 in Uppsala, Sweden.

The results of the algorithm combining active surfaces and volumetric FE models on intraoperative sequences showing only gravity-related brain shift (first experiment in Chapter 9) have been published (Ferrant et al., 2000c)
and presented at the *MICCAI 2000* conference in October 2000 in Pittsburgh, PA, USA.

An extended version combining these two last papers (Ferrant et al., 2000a,c), fully describing the algorithm as well as the brain shift application has been submitted as a journal paper to the *IEEE Transactions on Medical Imaging* in November 2000.

After MICCAI 2000, we were invited to submit a journal paper to Elsevier’s *Medical Image Analysis* journal. We submitted the results of the second experiment in Chapter 9 on serial registration of intraoperative brain images, with details about the resection modeling as well (Ferrant et al., 2001a).

Our efforts in optimizing the memory and computational requirements of our algorithms, as well as the parallelization of our algorithms have led to a publication (Warfield et al., 2000a) and a presentation by Simon Warfield at the *SC2000: High Performance Computing and Networking* conference in November 2000 in Dallas, USA.

The work I did for developing all the 3D (renderings, color-coded deformation fields, tensor visualizations, etc.) and 2D visualization tools (mainly for visualizing cuts through 3D models and image data as well as 2D deformation arrows) that helped produce most of the figures throughout this thesis have been at the origin of yet another publication (Ferrant et al., 2001b) and presentation at the *SPIE Medical Imaging 2001* conference in February 2001 in San Diego, CA, USA.

Finally, the preliminary results on deformable atlas matching presented in Chapter 8 have recently been submitted to the *MICCAI 2001: Medical Image Computing and Computer Assisted Intervention* conference that will take place in October 2001 in Utrecht, The Netherlands.

### 10.4 Software Distribution and Collaborations

We developed our algorithms using a clear, well-commented style. Also, lots of efforts were made to optimize the code, both in terms of memory storage requirement and in terms of computational load. Special efforts were also made so that the code could run on parallel clusters or multi-
processor machines by using the publicly available MPI (Message Passing Interface) and PETSc (Portable, Extensible Toolkit for Scientific Computing) libraries. This has resulted in easily compilable and usable code, that we made available for research purposes.

The medical doctor that first used code I wrote was Arya Nabavi, who is a neurosurgeon. He insisted to try out the elastic image matching code of the algorithm presented in Chapter 5 on intraoperative MR image sequences of the brain. His attempts to run the code were rather successful and the results presented in section 7.1.4 of Chapter 7 and were used in a publication that will appear in the *Neurosurgery* journal (Nabavi et al., 2001).

The first person to use the mesher code and the FE module for solving elasticity problems was Filip Schutyser, who is doing maxillo-facial surgical planning. He uses the model to do soft tissue deformation predictions. For the use of the software and the help I provided him, I was granted coauthorship on his last publication (Schutyser et al., 2000).

Another person working on intraoperative deformable registration for prostate surgery was Aditya Bharatha from Canada. He spent the summer months of 2000 to get results for his project on prostate registration using the algorithm presented in Chapter 6. The algorithm was successfully applied to about 15 cases, and a journal paper for which I have also been granted coauthorship was submitted to the *Journal of Medical Physics* (Bharatha et al., 2000).

All these successful collaborations and the feedback of the users have greatly contributed to improve the software. Simon Warfield has been following the code very closely, and much of the coding style I acquired derives from his experience. Setting up a CVS ( Concurrent Versions System) repository that centralized changes other people wanted to make to the software has been very helpful too. The PETSc team also provided a lot of technical help for parallelizing our code in conjunction with their library. Other people at SPL and MIT have provided valuable feedback for improving the software. The code currently still is available, and we have requests from all over the world.
Bibliography


Bibliography


List of Figures

1.1 View of the operating room during neurosurgery with an intraoperative MR scanner. ................................. 3

1.2 Coronal view of successive intraoperative MR scans illustrating brain deformation during neurosurgery. a) Before start of surgery, b) after opening of the dura, c) after start of tumor resection. .......................................................... 4

2.1 a) 2D Triangular FE mesh. b) Linear shape function of node i of the triangular element. ................................. 15

2.2 Timing results for assembling, solving, and the sum of initialization, assembling and solving times of 77511 equations simulating the FE elastic deformation of a brain on the Deep-flow cluster. .......................................................... 18

2.3 Deformation of an isotropic elastic cube ($E = 1 \ kPa$, $F = 1000 \ N$ ; (a,b) $\nu = 0$ ; (c,d) $\nu = 0.4$). (a,c) View of the deformed outer surface. (b,d) View of orthogonal cuts through deformed volume. For $\nu = 0$, the cube is linearly compressed and roughly retains its shape. For $\nu = 0.4$, the nearly incompressible material causes large shape changes and lateral expansion of the cube toward the bottom. Color-coding corresponds to the intensity of the deformation in mm. The pink wireframe is that of the original cube. ................................. 19

3.1 Different basic configuration cases for marching cubes iso-surfacing. ......................................................... 23

3.2 Different basic configuration cases for Marching Tetrahedra iso-surfacing. ................................................. 24

3.3 Block scheme of multi-resolution mesh generator. ............ 27

3.4 Two possible subdivisions of a cube into five tetrahedra. .... 29

3.5 Subdivision of a cube into six tetrahedra. ......................... 30
3.6 Different subdivision of a tetrahedron given edge splittings. 31
3.7 Different subdivisions of a prism given the quadrilateral faces’ diagonals. ........................................ 32
3.8 Different subdivisions of a tetrahedron given the nodes’ label configuration. ........................................ 32
3.9 Tetrahedral meshes of an embedded sphere. (a,b,c) Initial subdivision of image into 9x9x9 cubes, followed by clipping of the sphere. The boundary sphere has limited resolution, but most elements have a good aspect ratio. (c,d,e) Multi-resolution subdivision of initial cube of size 63x63x63 (entire volume) until smallest edge size was 2, followed by clipping. The boundary surface is more precise, but the shape of several elements is degraded. (a,d) Cut through tetrahedral mesh overlayed on corresponding cut through image. (b,e) 3D renderings of wireframe of entire mesh. (c,f) 3D Surface renderings of boundary surface of the sphere. ............. 33
4.1 a,b,c) 3D Surface renderings of initial, affine transformed and deformed surfaces. d,e,f) Cuts through initial, affine transformed and deformed surfaces overlayed on corresponding cut through target image. The affine transform captures global differences (translation, scaling), while the active surface deformation accounts for local shape differences. ....... 41
4.2 a,b) Cuts through the initial and deformed surfaces overlayed on corresponding cut through target image. c) 3D Surface rendering of target surface after active surface deformation. 42
5.1 Growing sphere. a) and b): close-ups of 2D cuts through 3D image with a) classical OF, and b) FE matching deformation fields overlayed, c) and 3D orthogonal cuts through the FE mesh with intensity coding of the displacement field. The displacement field is mainly located at the boundaries of the sphere and is propagated through the surrounding elastic medium, whereas in the OF setting it is only located at those locations in the image where the images differ. .............. 47
5.2 Sphere to cube experiment. 2D deformation field overlayed on cut through target and deformed images at 4 successive iterations of the matching process. The resulting match illustrates the accuracy of the match and the distribution of the deformation field over the volume. .................. 48
5.3 Sphere to cube experiment. Cut through deformed mesh overlayed on corresponding cut through target image. This illustrates how the deformation of the boundary of the sphere elastically stretches and compressed the volume into which the sphere is embedded.

6.1 Block schema of the deformable registration algorithm illustrating the matching of one image onto a target image.

6.2 Translated cube experiment. a) Cut through initial FE mesh overlayed on corresponding cut through image the mesh was generated from. b) Cut through initial boundary surface overlayed on corresponding cut through target image. c) Cut through deformed initial boundary surface (after affine transformation) overlayed on corresponding cut through target image. d) Cut through deformed FE mesh overlayed on corresponding cut through target image. This illustrates the rigid translation of the cube, pulling tissue of the upper left corner and compressing the soft tissue in the lower right corner.

6.3 Translated cube experiment. a) 2D deformation field overlayed on cut through initial image. b) 3D rendering of cuts through deformed FE mesh with arrows representing the actual 3D deformation field. Color-coding corresponds to the intensity of the deformation. The deformation field is translational within the hard cube, and has a rotational component in the soft part towards the edges of the image boundaries.

6.4 a) Slice 30 of the target image with a cut through the initial surface of the object overlayed. b) 3D surface rendering of the initial surface. c) The same slice with a cut through the deformed surface. d) 3D surface rendering of the deformed surface.

6.5 a) Orthogonal cuts through the initial volumetric mesh with the sphere extracted and b) the same with deformed mesh. These visualizations illustrate how the squeezing of the sphere onto the ellipsoid affects the surrounding elastic medium.

6.6 Illustration of the results of squeezing a sphere onto an ellipsoid. a) Axial cut through original mesh overlayed on slice of original image and b) the same with deformed mesh on target image. c) The deformation field overlayed on slice of target image.
7.1 Arm exercise analysis. Slice of 3D MR dataset a) at exercise, b) at rest, c) deformation field overlayed on exercise slice, indicating deformation mainly in the area where the muscle was exercised. 62

7.2 Illustration of the effect of enlarging ventricles of an MS patient. a) slice of difference between segmented images at both time points (gray means no difference), b) deformation field superimposed on same image at the first time point, c) close-up of b). 62

7.3 Matching of MS lesion growth. Close-up on slice of T2 weighted 3D MR image a) at time point 1, b) at time point 2, c) deformation field overlayed on time point 1. 63

7.4 3D Elastic matching of intraoperative MR images of the brain showing the propagation of the deformation at the brain surface throughout the volume. a) Axial slice of early intraoperative scan with deformation field resulting from the matching onto the later scan, b) same slice of the later scan, c) same slice of difference image. 64

7.5 a) Axial slice of early intraoperative scan, b) same slice of later intraoperative scan, after tumor resection. a to b) Result of matching brain tissue in image a onto brain tissue in image b. The deformation field illustrates the behavior of the brain tissue in the vicinity of the tumor after resection. 64

8.1 Slice of original MR image, segmentation using directional watersheds, brain contours, simplified contours. 72

8.2 First row : cuts (a:axial - b:saggital) through deformable surfaces after 2nd degree polynomial transform overlayed on corresponding cuts through target image. Second row : same cuts (c:axial - d:saggital) but after active surface deformation. 73

8.3 Volumetric deformations of the atlas MR image illustrating the transformation steps of the algorithm. a) Slice of deformed volume after second degree registration of ventricles and brain surface. b) Same slice after active surface deformation. c) Same slice of target image. 74

8.4 Deformation of other atlas objects overlayed on target image after volumetric transformation. 74
9.1 Typical sequence of intraoperative MR scans showing brain shift due to gravity and CSF draining. The second scan was taken just after dura removal, the third scan was taken after the brain had shifted.  

9.2 Axial slice through tumor and right lateral ventricle in each of five intraoperative MR scans taken sequentially during neurosurgery. The first scan (a) was taken before start of surgery, the second one (b) after opening the dura, the third and fourth ones show 2 stages of tumor resection (c,d) and the fifth (e) was acquired after dural closure.  

9.3 Block scheme for our serial intraoperative deformable tracking and registration algorithm.  

9.4 Axial, sagittal, and coronal cuts through tetrahedral mesh of the brain with refinement in the vicinity of the lateral ventricles overlayed on corresponding cuts through preoperative image.  

9.5 Illustration of the capture of the boundary surface deformation using our active surface algorithm. Axial cut through initial (a,b) and deformed (c) active surfaces overlayed on corresponding slice of a) initial; b,c) target intraoperative MR image.  

9.6 Results of the intraoperative volumetric update of preoperative image data. Axial slice of a) initial scan b) target scan c) initial scan deformed using our algorithm d) difference between target scan and deformed initial scan.  

9.7 Results of boundary surface deformation. 3D surface renderings of active surfaces with color-coded intensity of the deformation field. a) brain surface, b) lateral ventricles.  

9.8 Illustration of the propagation of the boundary surface deformation through the elastic brain volume. 3D Volumetric Deformation field (downsampled 12x, scaled 2x) with orthogonal cuts through target intraoperative MR image and transparently overlayed color coded intensity of the deformation field. a) Axial view, gravity is downwards. b) Coronal view, gravity goes from left to right.
9.9  Assessment of the accuracy of the registration algorithm using landmarks. a) Volumetric deformation field and initial landmarks (green) overlayed on initial intraoperative image slice. b) Same slice of deformed initial image with deformed initial landmarks (red). c) Same slice of target image with deformed landmarks.  

9.10  Illustration of intraoperative segmentations. Axial slice as in Figure 9.2 showing intraoperative segmentations of the scans taken during surgery.  

9.11  Orthogonal cuts through the tetrahedral mesh model overlayed on corresponding cuts through the gray-scale MR image.  

9.12  Axial slice and corresponding cuts through deformable boundary surface . From left to right, top to bottom : (a) 1st scan with initial boundary surfaces, (b) 2nd scan with same surfaces, (c) 2nd scan boundary surfaces deformed onto scan 2, (d) 3rd scan with same surfaces, (e) 3rd scan with deformed surfaces, (f) 4th scan with same surfaces, (g) 4th scan with deformed surfaces, (h) 5th scan with same surfaces, (i) 5th scan with deformed surfaces.  

9.13  3D surface renderings of the cortical surfaces with color coding of the deformation and 3D arrows showing the actual deformation from one scan to the next. a) represents the brain deformed onto scan 2 with deformation field from the 1st scan to the 2nd, b) brain 3 with deformation field from scan 2 to 3, c) brain 4 with deformation field from scan 3 to 4, d) brain 5 with deformation field from scan 4 to 5, e) brain 4 with deformation field from scan 4 to 5 (arrows are scaled 2 times here).  

9.14  3D surface renderings of the lateral ventricular surfaces with color coding of the deformation and 3D arrows showing the actual deformation from one scan to the next.  

9.15  2D axial (a,b) and coronal (c) cuts through topologically adapted boundary surface of the brain (red) and ventricles (yellow) overlayed on corresponding cut through 3rd MR scan.
9.16  a) 3D surface rendering of the brain surface after topological change modeling tissue resection. b) Surface rendering of the same surface cut along a sagittal plane showing the inside of the brain boundary surface at the level of the resection and tumor areas. ....................................................... 100

9.17  Axial slice (first row) and coronal slice (second row) of scans 3, 4, and 5 and corresponding cuts through deformable boundary surfaces (red: brain, yellow: ventricles). ............ 101

9.18  Orthogonal cuts along the coordinate axes of the color-coded FE model at the different imaged time points (2 to 5) during surgery transparently overlayed on corresponding cuts through intraoperative MR images. The arrows represent the deformation field, subsampled 5 times, and scaled 2 times. 102

9.19  Top row: 2D deformation field overlayed on axial slice at different time points (1 to 4) during surgery. Bottom row: enlargements of the same deformation field on the same slice. 103

9.20  Top row: enlargements around tumor and resection area of 2D deformation field from scan three to scan 4 on three slices (38, 43, and 47) of scan 3. Bottom row: same slices, but on scan 4 with deformation field from scan 4 to scan 5. ......... 104

9.21  Top row: axial slice of deformed images. Bottom row: same slice of difference image of deformed with target images. .. 104

9.22  3D visualization of stress tensor data. The colorcoding corresponds to the largest eigenvalue of the stress tensor at every node of the mesh. The stress tensors are subsampled by a factor 10 and represented using ellipsoids whose axes are scaled using the eigenvalues of the stress tensor. .............. 105

9.23  Top row: axial slice of intraoperative MR images with manually placed landmarks. Bottom row: same axial slice of deformed images with deformed landmarks. ............... 106

9.24  A timeline of typical intraoperative image acquisition and analysis tasks. ................................................................. 108
List of Tables

9.1 Average distance in \textit{mm} between deformed landmarks and manually placed landmarks. . . . . . . . . . . . . . . . . . . 107