Phase-driven Finite Element Model for Spatio-temporal Tracking in Tagged Cardiac MRI

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Abstract. Relative motion of the left ventricle can be a useful indicator of the degree of contraction. MRI tissue tagging produces noninvasive markers within the muscle wall which can be used to measure motion and deformation. However, the widespread use of tissue tagging has been limited by time-consuming image post-processing. We describe a method for automatically calculating displacements in 2D tagged images by using quadrature filters to estimating the phase. Since the tag represents a repeating pattern, they have an intrinsic phase property which stays the same as they move. We fit a 3D finite element model to the phase data in from successive MR images through systole. The addition of a 3D model allows for 1) estimating 3D motion from multiple 2D tagged images and 2) smoothing of the phase estimates which are susceptible to image artifacts and noise. This technique is applied to data acquired from normal hearts and compared with results of semi-automatic tracking.

1 Introduction

The application of MRI to the heart has allowed for better assessment of left ventricular (LV) function [2]. Variation in signal intensities between muscle, fat, blood and air allow for visualization of the inner and outer walls, or endocardium and epicardium. In order to visualize motion within the muscle wall, a non-invasive tissue tagging technique, also known as SPAMM, has been developed in the last decade [3]. With this method, radio-frequency pulses are used to saturate parallel planes of issue prior to imaging. Images are acquired perpendicular to these planes, resulting in dark bands in the images. The dark bands represent actual tissue and, since the saturation lasts through systole, reveal myocardial motion as the heart contracts. Figs. 1 and 2 show representative short and long-axis images. In the short-axis images, 2 perpendicular sets of tagging planes can be used to lay down a tagging grid.

The addition of the tagging grid helps in analyzing LV function. The tags provide material markers which can be followed through the heart beat. From tag motion, twisting of the heart can be revealed [14]. In addition, since the tags reveal relative motion, strain can also be derived [24] and used as an indicator of contraction.

In this paper, we describe a method for tracking the tags in successive image frames, by estimating the displacement at a pixel in two successive frames. Displacement is derived from the local phase, a technique previously used for estimating disparity between
two images in stereo vision and velocity measurements [10, 21]. Since the tags form a repetitive pattern in the images, the local phase is a material property and can be tracked. Another reason for using phase is stability: phase is independent of signal magnitude, which makes it insensitive to illumination variation over an image scene [9]. In MR this translates to insensitive to bias field inhomogeneities in the data. In contrast, a tracking system based on gradient magnitude will be sensitive to such variation.

One drawback of using 2D images is that they do not capture through-plane motion. Three-dimensional motion can be derived by combining information from tagged images taken from multiple views. We describe a finite element-based technique for 3D motion reconstruction. This is an extension of a deformable modelling technique which was previously applied to extracted tag data [13].

We here describe modified geometry and constraints which allows us to incorporate phase-based displacement estimates into the 3D motion reconstruction. As can be seen in the images, the tags at end systole vary smoothly from the endocardium to the epicardium. The 3D model ensures spatio-temporal smoothness via the finite element stiffness. This approach allows for stabilization the phase-derived displacement estimates, which are susceptible to noise and image artifacts. The technique is unique in that it combines phase-based displacement estimates with a deformable model and it is automatic.

2 Related Work

Several researchers have approached the problem of tracking tags in 2D images. The methods include optical flow [11, 7], template matching [8, 12] and least-squares minimization [12], Fourier transformation methods [25] and active contours [24]. Another group have developed a method which calculates strain directly from phase images [18].

Both model-based [17, 5, 19, 16, 22, 20] and non-model based [15, 6] approaches have been applied to reconstructing the 3D motion from multiple views. Some tech-
Fig. 2. Long-axis images at (a) beginning systole (b) mid-systole (c) end-systole

Techniques are applied directly to the images [17, 22, 20], while the rest operate on tag data extracted from the 2D images using one of the methods described above. Model-based approaches to 3D motion reconstruction include: fitting a 3D displacement field composed of an analytic series [17] or to predefined functions derived from known LV motion [5, 19] to data; using spline curve and surface reconstructions of tag data as input to a linear optimization procedure [16]; fitting of a cubic B-spline model [20] and nonlinear least-squares fitting of a finite element model [23].

3 Methods

3.1 Image Pre-processing

In an ideal sinusoidal signal, the phase is the argument of the analytical signal. In a more complicated signal, the local phase is the argument of the output of convolving it with complex quadrature filters. Since for a sinusoidal pattern the phase uniquely defines a spatial position modulo the wavelength, this phase can be used for estimating local displacements, assuming displacements smaller than the wavelength of the pattern.

The spherically separable quadrature filters are defined as:

$$Q(u) = R(\rho)D_k(\hat{u})$$

where $u$ is the vector valued frequency variable, $\rho = |u|$. The directional function, $D_k(\hat{u})$, is oriented along the direction of the tags. The radial frequency function $R(\rho)$ is a Gaussian function on a logarithmic scale, called a lognormal function:

$$R(\rho) = e^{-\frac{\ln^2(\rho/\rho_0)}{2B^2}}$$

is the radial frequency function. $B$ is the relative bandwidth in octaves and $\rho_0$ is the center frequency of the filter. $R(\rho)$ defines the frequency characteristics of the quadrature filters.
The tag spacing, \( t_s \) is known at the time of imaging or can be measured from the image. For each location and tag direction, two successive images are considered. For each pair \( I_i, i = 1, 2 \), the images are first convolved with the quadrature filter: \( s_i = Q(u) * I_i \).

The phase difference \( \Delta \phi \) is calculated as:

\[
\Delta \phi = \arg(s_1 * s_2). \tag{3}
\]

Also, the magnitude of \( s_1 * s_2 \) provides a measure for the certainty of the estimate. The phase disparity is used to estimate displacement, \( \beta \):

\[
\beta = \frac{\Delta \phi}{2\pi/t_s} \tag{4}
\]

### 3.2 FEM Deformable Model

We incorporate smoothing by fitting a solid finite element model to the displacement data. The model is initially a parallelepiped block, with elements aligned with the tag planes. During the fitting, elements for which the phase estimate is uncertain are eliminated.

**Dynamics** The energy formulation for the dynamics of a deformable solid dictates that for small displacements, the energy from external forces is absorbed by inertial, viscous, and the elastic energy of the solid itself [4]:

\[
\int \int \int \mathbf{F}_b \mathbf{u} \, dV + \int \int \mathbf{T} \mathbf{u} \, dS = \int \int \int \frac{1}{2} \varepsilon^T \mathbf{E} \varepsilon \, dV \tag{5}
\]

where the external and internal energy terms are on the and and right hand sides, respectively. The displacement and strains fields are, \( \mathbf{u} \) and \( \varepsilon \), respectively. \( \mathbf{F}_b \) contains body forces, \( \mathbf{T} \) contains surface tractions, \( \rho \) is the density of the material, and \( \kappa \) is the damping parameter.

For each element, the displacement field and its derivatives are approximated as

\[
\mathbf{u} = \mathbf{N}(\varepsilon, \eta, \psi)^T \mathbf{q} \tag{6}
\]

where \( \mathbf{q} \) are the displacements at the nodes of the mesh, and \( \mathbf{N} \) are the interpolation, or shape functions. These functions have as dependent variables the non-stationary, local element coordinate system \((\varepsilon, \eta, \psi)\). The shape functions can be seen as weighting the value (i.e. displacement) global coordinate of a node according to where a point lies within the \((\varepsilon, \eta, \psi)\) system of the element. Since \( \mathbf{N} \) are only dependent only position the derivatives are simple to calculate. We use eight-noded parallelepiped elements, whose linear shape functions are given in [4].

The discretized strain field is a function of discretized displacements:

\[
\varepsilon = \mathbf{B} \mathbf{d}, \tag{7}
\]
where $\mathbf{B}$ is derived by 1) converting strain to a sum of partial derivative of displacements; 2) using the inverse of the Jacobian to relate derivatives in the global coordinate system to those in the local element coordinate system, and 3) using the derivative of the shape functions to relate these local derivatives to nodal displacements, $\mathbf{d}$.

Eqs. 5 and 7 can be combined to obtain the following:

$$\mathbf{M}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} + \mathbf{K}\mathbf{q} = \mathbf{f}_q$$

(8)

where the vector the vector $\mathbf{f}_q$ contains the image-derived forces. The inertial term is neglected and the viscous damping (with a coefficient of one) is used in order to allow the model to come to rest when it reaches the position most consistent with the data. Stiffness can also be viewed as an internal resistance which results in a force equal to $\mathbf{K}\mathbf{q}$. Using these considerations, the equations of motion is written for each node, $i$, as

$$\mathbf{q}_i = \mathbf{f}_{i,\text{internal}} + \mathbf{f}_{i,\text{external}}$$

(9)

where $\mathbf{q}_i$ is the 3D nodal displacement, $\mathbf{f}_{i,\text{internal}} = [\mathbf{K}\mathbf{q}]_i$ and $\mathbf{f}_{i,\text{external}}$ is an external, image-derived force. It is important to note that these forces are only used to deform our model and are not meant to replicate the actual forces or material properties of cardiac tissue.

For each iteration, the external forces are calculated, the internal element forces are calculated and distributed to the nodes, and finally the nodal equations are solved using adaptive Euler integration.

**Constraints** During the image pre-processing, the 3D field of displacements for 3 different directions has been derived from both short and long-axis images. Prior to fitting between a pair of image frames, a target displacement, $\beta_i$, is assigned to each node by interpolating from the displacement field data. During each iteration, the forces on the nodes are set to be equal to the target displacement subtracted by the nodal displacement from previous iterations:

$$\mathbf{f}_{i,\text{internal}} = \beta_i - \mathbf{q}_i$$

(10)

The fitting continues until all forces are minimized, or the nodal displacement is approximately equal to the target displacement.

4 Results

The local phase was calculated and the difference was used to calculate the displacement. Each row of Fig. 3 shows the phase pairs and resulting displacement images.

In order to test the phase-driven deformable modeling technique, we applied it to simulated data. Given a cylindrical geometry and displacement field, we generated synthetic image data. The geometry and motion was similar to that found in the heart. The difference between reconstructed and prescribed tag locations was $0.8\text{mm}$ rms while the difference in nodal deformations was $0.61\text{mm}$, which are less than the pixel resolution.
Fig. 3. Pairs of phase images and displacement derived from them. Locations are same as those shown above (1st 2 rows = short-axis, 3rd row = long-axis).

Image data from normal patients was also analyzed using the method. Fig. 4 shows the intersection of the reconstructed model with the original image plane locations, along with the image. When these results were compared with those found using a semi-automatic tracking technique [1] an average discrepancy of 0.54mm was calculated between the two data sets.

5 Discussion

We describe a 3D tracking method in which phase-derived displacements are used as input into a deformable model. The model allows for the regularization of the phase estimates and integration of motion information from multiple image planes. This technique can easily be extended to 3D tagged images, where the model would serve as a regularizing force.
Fig. 4. Motion reconstruction results overlayed on original images. Representative a) short-axis and b) long-axis images.

References