Abstract
In this work we use a method based on mass conservation of blood flow to reduce the impact of noise and partial volume averaging in Phase Contrast Angiography. In such data, partial volume artifacts are a serious problem for small vessels. We show how the flow in small vessels can be greatly improved and better delineated. The mass conservation constraint is modeled using a Poisson equation and solved using a finite difference method.

Introduction:
Blood flow in vessels is considered to be a low speed flow, which can be assumed to be incompressible. The mass conservation constraint on a flow field, with neither sources nor sinks, simplifies to the following equation [1],
\[ \nabla \cdot \mathbf{u} = 0, \]
where \( \mathbf{u} \) is the velocity vector. The velocity field obtained by MRI is not guaranteed to satisfy the mass conservation equation (1), since the MRI data may contain errors, such as artifacts, phase wrapping, etc. The correction of the experimental flow field data, using computational fluid dynamics, is one of the most active research areas in the field of flow visualization for engineering applications, such as particle image velocimetry (PIV) [2,3].

The objective of this work is to modify the MRI velocity data by imposing the mass conservation equation (1), with the assumption of steady state flow and rigid vessel walls. The expected results are both the improvement of the connectivity of vessels, and the reduction of random noise.

Methods:
We impose the mass conservation constraint on the MRI velocity data, \( \mathbf{u}_{\text{MRI}} \), by introducing the velocity potential \( \phi \). The velocity field, \( \mathbf{u}_{\phi} \), generated by this potential, is simply written as [1]
\[ \nabla \phi = \mathbf{u}_{\phi}. \]

We create the new modified velocity field by adding \( \mathbf{u}_{\phi} \) to \( \mathbf{u}_{\text{MRI}} \). By substituting this new velocity field into Eq. (1), we obtain the equation for the velocity potential \( \phi \), which should satisfy the following Poisson equation,
\[ \Delta \phi = -\nabla \cdot \mathbf{u}_{\text{MRI}}. \]

Equation (3) is discretized by finite differences and solved by the iterative method. We make use of a staggered grid coordinate system for the discretization, where the velocity is defined on the center of the grid, and the potential is on the corner of the grid.

We use an 8 point-stencil to calculate both the gradient in each direction and the divergence of the velocity. The discretized Poisson equation is evaluated by using the 27 point-stencil finite difference [4].

The most general solution of Eq. (3) has the following form
\[ \phi = c^{(0)} + c^{(1)}_1 x_1 + c^{(2)}_{ij} x_i x_j + c^{(3)}_{ijk} x_i x_j x_k + \ldots, \]
where \( c^{(n)} \)'s are \( n \)-th order constant coefficient tensors. It is important to note that a solution consisting of the first and the second terms of the RHS of Eq. (4) always satisfies Eq. (3), regardless of the value of \( c \)'s. The first term generates zero velocity, while the second term generates a uniform velocity field.

Results:
We tested our method on a synthetic pipe flow with a discontinuity in the middle of the pipe, for which velocity vectors are shown on Fig. 1(a). The whole computational domain was 16^3 grid points. The pipe radius was 3, the initial flow field was set to be a Poiseuille flow. Figure 1(b) shows the results after correction. Clearly, improvement of the connectivity is observed. It is also confirmed that the restoration of the discontinuity is followed by the generation of small fluctuations, which can be seen in the neighborhood of the original discontinuity. This is mainly because our method is a global one, involving the solution of an elliptic equation. However, it can be stressed that the magnitude of the fluctuation generated by this method is much smaller than those of the "real" velocities.

Discussion:
A novel flow image correction method for MRI flow velocity field data was developed by imposing the mass conservation constraint, which is the fundamental condition for incompressible fluid flow. The improvement of the connectivity of the vessel tube was clearly confirmed on the MRI data. The upper left picture in Fig. 2 is the magnification of a part of the original data, and the right one is the corrected result. It is clearly observed that the image contrast of the very small vessels is much improved.

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References: