Level Set Based Integration of Segmentation and Computational Fluid Dynamics for Flow Correction in Phase Contrast Angiography

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Abstract. A novel approach to correct flow data from phase contrast angiography (PCA) is presented. The method is based on combining computational fluid dynamics (CFD) and segmentation in a level set framework. The MR-PCA velocity data is used not only a partial differential equation (PDE) based fast local level set for vessel segmentation, but also in a level set equation solving for a physically meaningful flow. The second level set is based on the ghost fluid method, where the data defines initial and boundary conditions. The segmentation and CFD systems are integrated to provide simultaneously a robust method yielding a physically correct velocity and an optimal vessel geometry. The application of this system to both synthetic and clinical data are shown and validity is discussed.

1 Introduction

In 3D phase contrast angiography (PCA) sequences, the velocities of blood flow in three orthogonal directions are mapped to phase differences, controlled by a variable known as the velocity encoding or venc [1]. This sequence results in phase wrapping, in area of flow with greater speed than the venc. Also turbulence and vortex, due to pulse or branch, can degenerated the signal quality, since they promote phase dispersion. These artifacts cause problems in flow quantification, especially in determining the flow direction. Another problem arises in MR-PCA when vessel diameter should be determined, since the MR signals in the neighborhood of vessel wall are very weak. This leads to the degeneration of the signal resolution near the wall.

Computational approach base on MR segmentation has been applied in arterial biomechanics [2], hemodynamics of carotid artery bifurcations [3], and with level set methods [4]. A combined computational fluid dynamics (CFD) and MRI studies have been conducted on the reconstruction of blood flow patterns in a human carotid bifurcation [5]. These studies are generally employing complicated unstructured CFD grid system, constructed from medical images [6]. However, these computational studies for the blood flow in vessels use MRI data only for
the segmentation, i.e., grid system generation. MR-PCA flow velocity data is not properly employed for most CFD studies of blood flow.

We developed a numerical code to simulate blood flow in vessel, by solving incompressible Navier-Stokes equation with vessel geometry segmented by partial differential equation (PDE) based fast local level set method [7]. This approach made it possible to enforce a zero-velocity boundary condition on vessel wall, which is zero-level, without smearing physical properties near the wall, by implement of level set ghost fluid method (GFM) concept [8]. This enable us to use simple structured computational grid, even same as those of MR-PCA. The improvement of velocity field is verified for both synthetic and clinical data.

2 Numerical Formulation

We develop a numerical scheme to simulate incompressible fluid flow in a tubular flow path, bounded with the solid rigid wall, in order to study blood flow in stationary vessel. Our approach is to consider the solid tubular wall boundary as the interface between the incompressible fluid and the rigid solid, then the stational interface problem is solved by applying the level set methods, which is the most popular scheme to solve interface problems. The level set method was originally developed by Osher and Sethian [9] as a simple and versatile method for computing and analyzing the motion of an interface in two or three dimensions, especially, for computing two-phase Navier-Stokes incompressible flows [10]. However, the original level set method cannot avoid from smearing out both the density and the viscosity across the interface, in order to prevent the spurious oscillatory solutions at the interface. As explained in [11], The original GFM was developed to solve this problem by populating cells next to the interface with “ghost values”, with extrapolated values across the interface.

We couple the incompressible Navier-Stokes equation solver with high accuracy, employing the level set scheme with projection method developed by Sussman et.al. [12], to the Ghost Fluid Method developed by Fedkiw et.al. [8], which resulted in success of stable enforcement of zero-velocity boundary condition on vessel wall.

2.1 Governing Equations

At each time step, we solve the following dimensionless evolution equations for the velocity and pressure,

\[ \nabla \cdot \mathbf{u} = 0 , \]

\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot (\nabla \mathbf{u}) = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} , \]

where \( t \) is time, \( \mathbf{u} \) is velocity, \( p \) is pressure. The dimensionless parameter, \( Re \), used in Eq. (2) is the Reynolds number (\( Re = \rho L U / \mu \)), where \( L \) and \( U \) are the characteristic length and velocity, respectively, \( \rho \) and \( \mu \) are density and viscosity of blood, respectively. We used the values for \( \rho \) as \( 1.055 \times 10^{-3} \) kg/m\(^3\), and \( \mu \) as \( 4.50 \times 10^{-3} \) kg/m s.
2.2 Discretization and Time Integration

We follow the discretization methodology and time integration procedure which Sussman, et.al. developed [12]. This scheme employs the third order essentially non-oscillatory method to evaluate convection term and the fractional time step projection method to enforce the continuity equation (1) to be satisfied. These methods guarantee the stability in high velocity field, the robustness in the complicated geometry, and the higher accuracy without smearing the solution.

2.3 Solid Wall Conditions using Ghost Cells

The ghost cells are defined in the solid side neighborhood of the fluid-solid interface (i.e. vessel wall) [8]. We can modify pressure in the ghost cells by using the isobaric fix technique [8], by defining the unit normal at every grid point as \( \mathbf{N} = \nabla \phi / |\nabla \phi| \) and then solving a partial differential equation for “constant extrapolation” in the normal direction. The equation is

\[
\frac{\partial p}{\partial \tau} + \mathbf{N} \cdot \nabla p = 0 .
\]

We developed the zero-velocity fix on the solid wall, by simple extension of the isobaric fix technique. We consider the new variable \( v = v(\phi) \): First, the constant extrapolation of \( v \) is calculated in the direction of the normal to the solid wall in the neighborhood of the wall. Then zero velocity fix is completed by setting \( u = v \). The partial differential equation governing \( v \) to be solved is the same as Eq. (3). These equations need to be solved only for a few \( \tau \) steps to populate a narrow band of ghost cells.

2.4 Vessel Segmentation

We carried out vessel segmentation applying the PDE-based local level set method [7] to T1W MR-PCA blood velocity data. Reinitialization technique presented in [10], where the following Hamilton-Jacobi type equation;

\[
\frac{\partial d}{\partial \tau} + S(d_0)(|\nabla d| - 1) = 0 ,
\]

is solved to steady state, with the initial conditions;

\[
d(x, 0) = d_0(x) \begin{cases} 
-2.0 \Delta l & \text{if } ||u_m|| \geq \delta \\
2.0 \Delta l & \text{if } ||u_m|| < \delta 
\end{cases}
\]

where \( u_m \) is T1W MR-PCA velocity vector, \( \Delta l \) is the order of \( \Delta x \), and \( \delta \) is the threshold number. It is sufficient for the level set function (defined as the distance function) to be calculated in the narrow band [7].

It is observed that solving Eq. (4) with initial condition (5) provides with thinner vessel geometry, not only because MR-PCA signal tends to be significantly weak in the neighborhood of vessel wall, but also because it is unable to retain the initial position of the interface. We then resolve Eq. (4) with

\[
d(x, 0) = d_1(x) - \xi \cdot \Delta l
\]
as the initial conditions instead of Eq. (5), where $d_1$ is the solution of Eqs. (4) with (5) The choice of $\xi$ will be discussed in the later section, and typically 0.0 < $\xi$ < 1.0. The distance function $d$ obtained by this procedure is employed as the level set function $\phi$ determining the flow path geometry for the flow field calculation.

2.5 Boundary Conditions

Calculations using clinical data are run with a rectangular parallelepiped part extracted from the original MR-PCA data. On the surface of the calculation domain, we need to specify the boundary conditions of both velocity and pressure. The velocity boundary conditions are set to be equal to the MR-PCA velocity data. Pressure boundary conditions for the cross section with maximum inlet velocity are set to be zero. For the other inlet and all the outlet cross sections, the pressure gradient normal to the calculation boundary surface are set to vanish.

3 Results

3.1 Flow in a Tube: Poiseuille Flow

We first calculated the flow field in a straight tube with circular cross section of constant radius. If pressure gradient along the tube is constant and known, the flow is known as a Poiseuille flow [14]. We chose this flow in order to verify the validity of the zero-velocity fix procedure on the solid wall.

![Fig. 1. Comparison between theoretical and numerical results for Poiseuille flow](image)

Figure 1(a) shows the comparison of velocity distribution between the theoretical (solid line) and the numerical (open circles) results. It can be clearly observed that both of them agree well, and especially that enforcement of the zero-velocity condition on the tube wall is verified to be accurate. These results strongly suggest that the treatment of both isobar and zero-velocity fixes are valid and effective.

Figure 1(b) shows the pressure distribution along the tube. Our numerical scheme employs the zero gradient condition for the outlet pressure, hence the
pressure gradient cannot be constant. Therefore the discrepancies from theoretical result are inevitable. However, since these discrepancies are sufficiently small, we can verify that our method works well in simulation of the flow with tubular geometry.

We next tested our method with both boundary and initial conditions contaminated with noise. Figure 2(a) shows the synthetic velocity field generated by adding Gaussian white noise to the three components of the velocity field. Figure 2(b) shows the calculation result after 10 time steps. It can be clearly observed that the velocity field is quite improved with respect to the flow direction, except for the inlet and outlet boundaries where no improvement can be achieved with the current approach. Furthermore, the vessel boundary has not been altered as it would have with direct smoothing, for example.

![Image](initial_calculated.png)

(a) Initial  (b) Calculated

**Fig. 2.** Numerical simulation with contaminated initial condition for Poiseuille flow (left) and calculation result (right)

### 3.2 MR-PCA Data

In this section, we present results from applying our method to clinical data. The size of the data set are $256 \times 256 \times 60$, with field of view (FOV) of 240 mm, slice thickness of 1.5 mm, and velocity encoding of 40 cm/s. Figure 3 shows the MIP of this image.

![Image](mip.png)

(a) Original MR data  (b) Close up

**Fig. 3.** MIP of MR-PCA
After segmentation procedure described in the previous section, $\phi_\xi$ in Eq. (6) is calculated with various $\xi$. This parameter controls the vessel wall location. Then the velocity field, $u_\xi$, for a given $\phi_\xi$ and $u_M$ can be calculated. We assume that the most appropriate $\xi$, for a given set of MR-PCA velocity data, minimizes the discrepancy between the MR-PCA data and the calculation results. We employ the following expression for this discrepancy, $e$:

$$e = \sum ||u_M - u_\xi|| / \sum ||u_M||.$$  

The first data is a part of common carotid artery (shown as “A” in Fig. 3(b)). The flow field in a bending vessel geometry is calculated. This flow is chosen for the test of both stability and robustness of the method, since the geometry is rather simple to be segmented. In Table 1, the effect of the level set correction term, $\xi$, on the velocity calculations is shown. We chose $\xi = 0.6$, since the discrepancy $e$ is minimum, and calculation results are shown in Fig. 4.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>1.0</th>
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<td>$e$</td>
<td>0.580</td>
<td>0.534</td>
<td>0.516</td>
<td>0.490</td>
<td>0.468</td>
<td>0.451</td>
<td>0.456</td>
<td>0.469</td>
<td>0.502</td>
<td>0.738</td>
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**Fig. 4.** Comparison between MR-PCA and CFD velocity field for common carotid artery

It can be observed that flow field is significant improved, especially the direction of velocity vectors are naturally aligned along the vessel direction. Noticed also that speeds in the original data set, $||u_M||$’s, tend to be greater than those in the calculation, $||u_\xi||$’s, around both elbow and outlet regions. Considering the continuity equation (1), and the velocity distribution around elbow region shown in Fig. 4(a), it is most possible that the segmentation process provided an thicker vessel diameter around elbow region. This result also suggests that modification of outlet boundary condition may provide an improved velocity distribution.
The second data is the bifurcation region of basilar artery and vertebral arteries (shown as “B” in Fig. 3(b)). This is chosen for the test of more complicated flow than the previous one. The effect of the level set correction term, $\xi$, on the resulting flow is listed in Table 2. It should be emphasized that the optimal value of $\xi$ depends on both vessel geometry and the MR-PCA signal intensity distribution. The numerical results with $\xi = 0.2$ are shown in Fig. 5. Notice that the result is smooth and stable, even with the relatively small amount calculation points in our example.

<table>
<thead>
<tr>
<th>$\xi$</th>
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<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
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<td>$c$</td>
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<td>0.494</td>
<td>0.500</td>
<td>0.509</td>
<td>0.550</td>
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**Fig. 5.** Comparison between MR-PCA and CFD velocity field for bifurcation of basilar and vertebral arteries

Close to the bifurcation, erroneous velocity can be observed, possibly due to phase wrapping, in Fig. 5(a). These errors are successfully suppressed as shown in Fig. 5(b). Considering the strength of velocity from right vessel at the bifurcation, it is also possible that vessel diameter is overestimated, and this information can be employed for the re-segmentation of vessel.

## 4 Conclusion

A novel correction procedure of MR-PCA velocity data has been developed, by coupling incompressible Navier-Stokes equation solver with projection level set GFM, to a PDE-based fast local level set vessel segmentation method.
Applying this procedure to both synthetic and clinical data, significant improvement on the blood velocity field, such as smooth velocity distribution aligned along vessel, and removal of burst or error vectors, could be observed. This procedure also provides possibilities for improved vessel segmentation. The authors are aware of the necessity of more quantitative validation of this procedure, for example, flow phantom.

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References